

# **KNOWLEDGE INSTITUTE OF TECHNOLOGY**

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*Beyond Knowledge*

## **QUESTION BANK**

### **Heat and Mass Transfer**

**Department of Mechanical Engineering**

## Unit-1 Conduction

### Part-A

#### 1. State Fourier's Law of conduction. (

The rate of heat conduction is proportional to the area measured normal to the direction of heat flow and to the temperature gradient in that direction.

$$Q \propto A \frac{dT}{dx} \quad Q = -KA \frac{dT}{dx} \quad \text{where } A - \text{are in } m^2$$

$\frac{dT}{dx}$  - Temperature gradient in K/m      K – Thermal conductivity W/mK.

#### 2. Define Thermal Conductivity.

Thermal conductivity is defined as the ability of a substance to conduct heat.

#### 3. Write down the equation for conduction of heat through a slab or plane wall.

$$\text{Heat transfer } Q = \frac{\Delta T_{\text{overall}}}{R} \quad \text{Where } \Delta T = T_1 - T_2$$

$$R = \frac{L}{KA} - \text{Thermal resistance of slab}$$

L = Thickness of slab,      K = Thermal conductivity of slab,      A = Area

#### 4. Write down the equation for conduction of heat through a hollow cylinder.

$$\text{Heat transfer } Q = \frac{\Delta T_{\text{overall}}}{R} \quad \text{Where, } \Delta T = T_1 - T_2$$

$$R = \frac{1}{2\pi LK} \ln \left[ \frac{r_2}{r_1} \right] \text{ thermal resistance of slab}$$

L – Length of cylinder, K – Thermal conductivity,  $r_2$  – Outer radius,  $r_1$  – inner radius

#### 5. State Newton's law of cooling or convection law.

Heat transfer by convection is given by Newton's law of cooling

$$Q = hA (T_s - T_\infty)$$

Where

A – Area exposed to heat transfer in  $m^2$ ,      h - heat transfer coefficient in  $W/m^2K$

$T_s$  – Temperature of the surface in K,       $T_\infty$  - Temperature of the fluid in K.

**6. Write down the general equation for one dimensional steady state heat transfer in slab or plane wall with and without heat generation.**

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\infty} \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{K} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

**7. Define overall heat transfer co-efficient.**

The overall heat transfer by combined modes is usually expressed in terms of an overall conductance or overall heat transfer co-efficient 'U'.

$$\text{Heat transfer } Q = UA \Delta T.$$

**8. Write down the equation for heat transfer through composite pipes or cylinder.**

$$\text{Heat transfer } Q = \frac{\Delta T_{\text{overall}}}{R}, \quad \text{Where, } \Delta T = T_a - T_b, \quad R = \frac{1}{2\pi L} \left[ \frac{1}{h_a r_1} + \frac{\ln \left[ \frac{r_2}{r_1} \right]}{K_1} + \frac{\ln \left[ \frac{r_1}{r_2} \right] L_2}{K_2} + \frac{1}{h_b r_3} \right].$$

**9. What is critical radius of insulation (or) critical thickness?**

$$\text{Critical radius} = r_c$$

$$\text{Critical thickness} = r_c - r_1$$

Addition of insulating material on a surface does not reduce the amount of heat transfer rate always. In fact under certain circumstances it actually increases the heat loss up to certain thickness of insulation. The radius of insulation for which the heat transfer is maximum is called critical radius of insulation, and the corresponding thickness is called critical thickness.

**10. Define fins (or) extended surfaces.**

It is possible to increase the heat transfer rate by increasing the surface of heat transfer. The surfaces used for increasing heat transfer are called extended surfaces or sometimes known as fins.

**11. State the applications of fins.**

The main applications of fins are

1. Cooling of electronic components
2. Cooling of motor cycle engines.
3. Cooling of transformers
4. Cooling of small capacity compressors

**12. Define Fin efficiency.**

The efficiency of a fin is defined as the ratio of actual heat transfer by the fin to the maximum possible heat transferred by the fin.

$$\eta_{\text{fin}} = \frac{Q_{\text{fin}}}{Q_{\text{max}}}$$

### 13. Define Fin effectiveness.

Fin effectiveness is the ratio of heat transfer with fin to that without fin

$$\text{Fin effectiveness} = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$$

Part -B

1. A wall is constructed of several layers. The first layer consists of masonry brick 20 cm. thick of thermal conductivity 0.66 W/mK, the second layer consists of 3 cm thick mortar of thermal conductivity 0.6 W/mK, the third layer consists of 8 cm thick lime stone of thermal conductivity 0.58 W/mK and the outer layer consists of 1.2 cm thick plaster of thermal conductivity 0.6 W/mK. The heat transfer coefficient on the interior and exterior of the wall are 5.6 W/m<sup>2</sup>K and 11 W/m<sup>2</sup>K respectively. Interior room temperature is 22°C and outside air temperature is -5°C.

Calculate

- Overall heat transfer coefficient
- Overall thermal resistance
- The rate of heat transfer
- The temperature at the junction between the mortar and the limestone.

Given Data

Thickness of masonry  $L_1 = 20\text{cm} = 0.20\text{ m}$

Thermal conductivity  $K_1 = 0.66\text{ W/mK}$

Thickness of mortar  $L_2 = 3\text{cm} = 0.03\text{ m}$

Thermal conductivity of mortar  $K_2 = 0.6\text{ W/mK}$

Thickness of limestone  $L_3 = 8\text{ cm} = 0.08\text{ m}$

Thermal conductivity  $K_3 = 0.58\text{ W/mK}$

Thickness of Plaster  $L_4 = 1.2\text{ cm} = 0.012\text{ m}$

Thermal conductivity  $K_4 = 0.6\text{ W/mK}$

Interior heat transfer coefficient  $h_a = 5.6\text{ W/m}^2\text{K}$

Exterior heat transfer co-efficient  $h_b = 11\text{ W/m}^2\text{K}$

Interior room temperature  $T_a = 22^\circ\text{C} + 273 = 295\text{ K}$

Outside air temperature  $T_b = -5^\circ\text{C} + 273 = 268\text{ K}.$

**Solution:**

Heat flow through composite wall is given by

$$Q = \frac{\Delta T_{\text{overall}}}{R} \text{ [From equation (13)] (or) [HMT Data book page No. 34]}$$

Where,  $\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}}$$

$$\Rightarrow Q/A = \frac{295 - 268}{\frac{1}{5.6} + \frac{0.20}{0.66} + \frac{0.03}{0.6} + \frac{0.08}{0.58} + \frac{0.012}{0.6} + \frac{1}{11}}$$

$$\boxed{\text{Heat transfer per unit area } Q/A = 34.56 \text{ W/m}^2}$$

We know, Heat transfer  $Q = UA (T_a - T_b)$  [From equation (14)]

Where  $U$  – overall heat transfer co-efficient

$$\Rightarrow U = \frac{Q}{A \times (T_a - T_b)}$$

$$\Rightarrow U = \frac{34.56}{295 - 268}$$

$$\boxed{\text{Overall heat transfer co-efficient } U = 1.28 \text{ W/m}^2 \text{K}}$$

We know

Overall Thermal resistance ( $R$ )

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}$$

For unit Area

$$R = \frac{1}{h_a} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{L_4}{K_4} + \frac{1}{h_b}$$

$$= \frac{1}{56} + \frac{0.20}{0.66} + \frac{0.03}{0.6} + \frac{0.08}{0.58} + \frac{0.012}{0.6} + \frac{1}{11}$$

$$\boxed{R = 0.78 \text{ K/W}}$$

**Interface temperature between mortar and the limestone  $T_3$**

Interface temperatures relation

$$\Rightarrow Q = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_5}{R_4} = \frac{T_5 - T_b}{R_b}$$

$$\Rightarrow Q = \frac{T_a - T_1}{R_a}$$

$$Q = \frac{295 - T_1}{1/h_a A} \quad \left[ \because R_a = \frac{1}{h_a A} \right]$$

$$\Rightarrow Q/A = \frac{295 - T_1}{1/h_a}$$

$$\Rightarrow 34.56 = \frac{295 - T_1}{1/5.6}$$

$$\Rightarrow \boxed{T_1 = 288.8 \text{ K}}$$

$$\Rightarrow Q = \frac{T_1 - T_2}{R_1}$$

$$Q = \frac{288.8 - T_2}{\frac{L_1}{K_1 A}} \quad \left[ \because R_1 = \frac{L_1}{K_1 A} \right]$$

$$\Rightarrow Q/A = \frac{288.8 - T_2}{\frac{L_1}{K_1}}$$

$$\Rightarrow 34.56 = \frac{288.8 - T_2}{\frac{0.20}{0.66}}$$

$$\Rightarrow \boxed{T_2 = 278.3 \text{ K}}$$

$$\Rightarrow Q = \frac{T_2 - T_3}{R_2}$$

$$Q = \frac{278.3 - T_3}{\frac{L_2}{K_2 A}} \quad \left[ \because R_2 = \frac{L_2}{K_2 A} \right]$$

$$\Rightarrow Q/A = \frac{278.3 - T_3}{\frac{L_2}{K_2}}$$

$$\Rightarrow 34.56 = \frac{278.3 - T_3}{\frac{0.03}{0.6}}$$

$$\Rightarrow \boxed{T_3 = 276.5 \text{ K}}$$

Temperature between Mortar and limestone ( $T_3$  is 276.5 K)

**2. A furnace wall made up of 7.5 cm of fire plate and 0.65 cm of mild steel plate. Inside surface exposed to hot gas at 650°C and outside air temperature 27°C. The convective**

heat transfer co-efficient for inner side is  $60 \text{ W/m}^2\text{K}$ . The convective heat transfer co-efficient for outer side is  $8 \text{ W/m}^2\text{K}$ . Calculate the heat lost per square meter area of the furnace wall and also find outside surface temperature.

### Given Data

Thickness of fire plate  $L_1 = 7.5 \text{ cm} = 0.075 \text{ m}$

Thickness of mild steel  $L_2 = 0.65 \text{ cm} = 0.0065 \text{ m}$

Inside hot gas temperature  $T_a = 650^\circ\text{C} + 273 = 923 \text{ K}$

Outside air temperature  $T_b = 27^\circ\text{C} + 273 = 300^\circ\text{K}$

Convective heat transfer co-efficient for

Inner side  $h_a = 60 \text{ W/m}^2\text{K}$

Convective heat transfer co-efficient for

Outer side  $h_b = 8 \text{ W/m}^2\text{K}$ .

### Solution:

#### (i) Heat lost per square meter area (Q/A)

Thermal conductivity for fire plate

$$K_1 = 1035 \times 10^{-3} \text{ W/mK} \quad [\text{From HMT data book page No.11}]$$

Thermal conductivity for mild steel plate

$$K_2 = 53.6 \text{ W/mK} \quad [\text{From HMT data book page No.1}]$$

Heat flow  $Q = \frac{\Delta T_{\text{overall}}}{R}$ , Where

$$\Delta T = T_a - T_b$$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}}$$

[The term  $L_3$  is not given so neglect that term]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}}$$

The term  $L_3$  is not given so neglect that term]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{1}{h_b A}}$$

$$Q/A = \frac{923 - 300}{\frac{1}{60} + \frac{0.075}{1.035} + \frac{0.0065}{53.6} + \frac{1}{8}}$$

$$\boxed{Q/A = 2907.79 \text{ W/m}^2}$$

(ii) **Outside surface temperature  $T_3$**

We know that, Interface temperatures relation

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_b}{R_b} \dots (A)$$

$$(A) \Rightarrow Q = \frac{T_3 - T_b}{R_b}$$

where

$$R_b = \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_3 - T_b}{\frac{1}{h_b A}}$$

$$\Rightarrow Q/A = \frac{T_3 - T_b}{\frac{1}{h_b}}$$

$$\Rightarrow 2907.79 = \frac{T_3 - 300}{\frac{1}{8}}$$

$$\boxed{T_3 = 663.473 \text{ K}}$$

**3. A steel tube ( $K = 43.26 \text{ W/mK}$ ) of 5.08 cm inner diameter and 7.62 cm outer diameter is covered with 2.5 cm layer of insulation ( $K = 0.208 \text{ W/mK}$ ) the inside surface of the tube receives heat from a hot gas at the temperature of  $316^\circ\text{C}$  with heat transfer co-efficient of  $28 \text{ W/m}^2\text{K}$ . While the outer surface exposed to the ambient air at  $30^\circ\text{C}$  with heat transfer co-efficient of  $17 \text{ W/m}^2\text{K}$ . Calculate heat loss for 3 m length of the tube.**

### Given

Steel tube thermal conductivity  $K_1 = 43.26 \text{ W/mK}$

Inner diameter of steel  $d_1 = 5.08 \text{ cm} = 0.0508 \text{ m}$

Inner radius  $r_1 = 0.0254 \text{ m}$

Outer diameter of steel  $d_2 = 7.62 \text{ cm} = 0.0762 \text{ m}$

Outer radius  $r_2 = 0.0381 \text{ m}$

Radius  $r_3 = r_2 + \text{thickness of insulation}$

Radius  $r_3 = 0.0381 + 0.025 \text{ m} \quad r_3 = 0.0631 \text{ m}$

Thermal conductivity of insulation  $K_2 = 0.208 \text{ W/mK}$

Hot gas temperature  $T_a = 316^\circ\text{C} + 273 = 589 \text{ K}$

Ambient air temperature  $T_b = 30^\circ\text{C} + 273 = 303 \text{ K}$

Heat transfer co-efficient at inner side  $h_a = 28 \text{ W/m}^2\text{K}$

Heat transfer co-efficient at outer side  $h_b = 17 \text{ W/m}^2\text{K}$

Length  $L = 3 \text{ m}$

### Solution :

Heat flow  $Q = \frac{\Delta T_{\text{overall}}}{R}$  [From equation No.(19) or HMT data book Page No.35]

Where  $\Delta T = T_a - T_b$

$$R = \frac{1}{2\pi L} \left[ \frac{1}{h_a r_1} + \frac{1}{K_1} \ln \left[ \frac{r_2}{r_1} \right] + \frac{1}{K_2} \ln \left[ \frac{r_3}{r_2} \right] + \frac{1}{K_3} \ln \left[ \frac{r_4}{r_3} \right] + \frac{1}{h_b r_4} \right]$$
$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{2\pi L} \left[ \frac{1}{h_a r_1} + \frac{1}{K_1} \ln \left[ \frac{r_2}{r_1} \right] + \frac{1}{K_2} \ln \left[ \frac{r_3}{r_2} \right] + \frac{1}{K_3} \ln \left[ \frac{r_4}{r_3} \right] + \frac{1}{h_b r_4} \right]}$$

[The terms  $K_3$  and  $r_4$  are not given, so neglect that terms]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{2\pi L} \left[ \frac{1}{h_a r_1} + \frac{1}{K_1} \ln \left[ \frac{r_2}{r_1} \right] + \frac{1}{K_2} \ln \left[ \frac{r_3}{r_2} \right] + \frac{1}{h_b r_3} \right]}$$

$$\Rightarrow Q = \frac{589 - 303}{\frac{1}{2\pi \times 3} \left[ \frac{1}{28 \times 0.0254} + \frac{1}{43.26} \ln \left[ \frac{0.0381}{0.0254} \right] + \frac{1}{0.208} \ln \left[ \frac{0.0631}{0.0381} \right] + \frac{1}{17 \times 0.0631} \right]}$$

$$\boxed{Q = 1129.42 \text{ W}}$$

Heat loss  $Q = 1129.42 \text{ W}$ .

### 4. Derive an expression of Critical Radius of Insulation For A Cylinder.

Consider a cylinder having thermal conductivity  $K$ . Let  $r_1$  and  $r_0$  inner and outer

radii of insulation.

$$\text{Heat transfer } Q = \frac{T_i - T_\infty}{\frac{\ln \left[ \frac{r_0}{r_1} \right]}{2\pi KL}} \quad [\text{From equation No.(3)}]$$

Considering h be the outside heat transfer co-efficient.

$$\therefore Q = \frac{T_i - T_\infty}{\frac{\ln \left[ \frac{r_0}{r_1} \right]}{2\pi KL} + \frac{1}{A_0 h}}$$

Here  $A_0 = 2\pi r_0 L$

$$\Rightarrow Q = \frac{T_i - T_\infty}{\frac{\ln \left[ \frac{r_0}{r_1} \right]}{2\pi KL} + \frac{1}{2\pi r_0 L h}}$$

To find the critical radius of insulation, differentiate Q with respect to  $r_0$  and equate it to zero.

$$\Rightarrow \frac{dQ}{dr_0} = \frac{0 - (T_i - T_\infty) \left[ \frac{1}{2\pi KL r_0} - \frac{1}{2\pi h L r_0^2} \right]}{\frac{1}{2\pi KL} \ln \left[ \frac{r_0}{r_1} \right] + \frac{1}{2\pi h L r_0}}$$

since  $(T_i - T_\infty) \neq 0$

$$\Rightarrow \frac{1}{2\pi KL r_0} - \frac{1}{2\pi h L r_0^2} = 0$$

$$\Rightarrow \boxed{r_0 = \frac{K}{h} = r_c}$$

5. A wire of 6 mm diameter with 2 mm thick insulation ( $K = 0.11 \text{ W/mK}$ ). If the convective heat transfer co-efficient between the insulating surface and air is  $25 \text{ W/m}^2\text{K}$ , find the critical thickness of insulation. And also find the percentage of change in the heat transfer rate if the critical radius is used.

**Given Data**

$$d_1 = 6 \text{ mm}$$

$$r_1 = 3 \text{ mm} = 0.003 \text{ m}$$

$$r_2 = r_1 + 2 = 3 + 2 = 5 \text{ mm} = 0.005 \text{ m}$$

$$K = 0.11 \text{ W/mK}$$

$$h_b = 25 \text{ W/m}^2\text{K}$$

**Solution :**

1. Critical radius  $r_c = \frac{K}{h}$  [From equation No.(21)]

$$r_c = \frac{0.11}{25} = 4.4 \times 10^{-3} \text{ m}$$

$$r_c = 4.4 \times 10^{-3} \text{ m}$$

$$\begin{aligned} \text{Critical thickness} &= r_c - r_1 \\ &= 4.4 \times 10^{-3} - 0.003 \\ &= 1.4 \times 10^{-3} \text{ m} \end{aligned}$$

$$\text{Critical thickness } t_c = 1.4 \times 10^{-3} \text{ (or) } 1.4 \text{ mm}$$

2. Heat transfer through an insulated wire is given by

$$Q_1 = \frac{T_a - T_b}{\frac{1}{2\pi L} \left[ \frac{\ln \left[ \frac{r_2}{r_1} \right]}{K_1} + \frac{1}{h_b r_2} \right]}$$

[From HMT data book Page No.35]

$$\begin{aligned} &= \frac{2\pi L (T_a - T_b)}{\left[ \frac{\ln \left[ \frac{0.005}{0.003} \right]}{0.11} + \frac{1}{25 \times 0.005} \right]} \\ Q_1 &= \frac{2\pi L (T_a - T_b)}{12.64} \end{aligned}$$

Heat flow through an insulated wire when critical radius is used is given by

$$Q_2 = \frac{T_a - T_b}{\frac{1}{2\pi L} \left[ \frac{\ln \left[ \frac{r_c}{r_1} \right]}{K_1} + \frac{1}{h_b r_c} \right]} \quad [r_2 \rightarrow r_c]$$

$$= \frac{2\pi L (T_a - T_b)}{\ln \left[ \frac{4.4 \times 10^{-3}}{0.003} \right] + \frac{1}{25 \times 4.4 \times 10^{-3}}}$$

$$Q_2 = \frac{2\pi L (T_a - T_b)}{12.572}$$

∴ Percentage of increase in heat flow by using

$$\text{Critical radius} = \frac{Q_2 - Q_1}{Q_1} \times 100$$

$$= \frac{\frac{1}{12.57} - \frac{1}{12.64}}{\frac{1}{12.64}} \times 100$$

$$= 0.55\%$$

6. An aluminium alloy fin of 7 mm thick and 50 mm long protrudes from a wall, which is maintained at 120°C. The ambient air temperature is 22°C. The heat transfer coefficient and conductivity of the fin material are 140 W/m<sup>2</sup>K and 55 W/mK respectively. Determine

1. Temperature at the end of the fin.
2. Temperature at the middle of the fin.
3. Total heat dissipated by the fin.

**Given**

Thickness  $t = 7\text{ mm} = 0.007\text{ m}$

Length  $L = 50\text{ mm} = 0.050\text{ m}$

Base temperature  $T_b = 120^\circ\text{C} + 273 = 393\text{ K}$

Ambient temperature  $T_\infty = 22^\circ + 273 = 295\text{ K}$

Heat transfer co-efficient  $h = 140\text{ W/m}^2\text{K}$

Thermal conductivity  $K = 55\text{ W/mK}$ .

**Solution :**

Length of the fin is 50 mm. So, this is short fin type problem. Assume end is insulated.

We know

Temperature distribution [Short fin, end insulated]

$$\frac{T - T_{\infty}}{T_b - T_{\infty}} = \frac{\cosh m [L - x]}{\cosh (mL)} \dots\dots (A)$$

[From HMT data book Page No.41]

**(i) Temperature at the end of the fin, Put  $x = L$**

$$\begin{aligned} (A) \Rightarrow \frac{T - T_{\infty}}{T_b - T_{\infty}} &= \frac{\cosh m [L - L]}{\cosh (mL)} \\ \Rightarrow \frac{T - T_{\infty}}{T_b - T_{\infty}} &= \frac{1}{\cosh (mL)} \quad \dots(1) \end{aligned}$$

where

$$m = \sqrt{\frac{hP}{KA}}$$

$$\begin{aligned} P = \text{Perimeter} &= 2 \times L \text{ (Approx)} \\ &= 2 \times 0.050 \end{aligned}$$

$$P = 0.1 \text{ m}$$

$$A - \text{Area} = \text{Length} \times \text{thickness} = 0.050 \times 0.007$$

$$A = 3.5 \times 10^{-4} \text{ m}^2$$

$$\Rightarrow m = \sqrt{\frac{hP}{KA}}$$

$$= \sqrt{\frac{140 \times 0.1}{55 \times 3.5 \times 10^{-4}}}$$

$$m = 26.96$$

$$(1) \Rightarrow \frac{T - T_{\infty}}{T_b - T_{\infty}} = \frac{1}{\cosh (26.9 \times 0.050)}$$

$$\Rightarrow \frac{T - T_{\infty}}{T_b - T_{\infty}} = \frac{1}{2.05}$$

$$\Rightarrow \frac{T - 295}{393 - 295} = \frac{1}{2.05}$$

$$\Rightarrow T - 295 = 47.8$$

$$\Rightarrow T = 342.8 \text{ K}$$

$$\text{Temperature at the end of the fin } T_{x=L} = 342.8 \text{ K}$$

**(ii) Temperature of the middle of the fin,**

Put  $x = L/2$  in Equation (A)

$$\begin{aligned}
 (A) \Rightarrow \frac{T - T_{\infty}}{T_b - T_{\infty}} &= \frac{\cosh m[L-L/2]}{\cosh mL} \\
 \Rightarrow \frac{T - T_{\infty}}{T_b - T_{\infty}} &= \frac{\cosh 26.9 \left[ 0.050 - \frac{0.050}{2} \right]}{\cosh [26.9 \times (0.050)]} \\
 \Rightarrow \frac{T - 295}{393 - 295} &= \frac{1.234}{2.049} \\
 \Rightarrow \frac{T - 295}{393 - 295} &= 0.6025 \\
 \boxed{T = 354.04 \text{ K}}
 \end{aligned}$$

Temperature at the middle of the fin

$$\boxed{T_{x=L/2} = 354.04 \text{ K}}$$

### (iii) Total heat dissipated

[From HMT data book Page No.41]

$$\begin{aligned}
 \Rightarrow Q &= (hPKA)^{1/2} (T_b - T_{\infty}) \tanh mL \\
 \Rightarrow [140 \times 0.1 \times 55 \times 3.5 \times 10^{-4}]^{1/2} \times (393 - 295) \\
 &\times \tanh (26.9 \times 0.050) \\
 \boxed{Q = 44.4 \text{ W}}
 \end{aligned}$$

**7. A copper plate 2 mm thick is heated up to 400°C and quenched into water at 30°C. Find the time required for the plate to reach the temperature of 50°C. Heat transfer co-efficient is 100 W/m<sup>2</sup>K. Density of copper is 8800 kg/m<sup>3</sup>. Specific heat of copper = 0.36 kJ/kg K.**

**Plate dimensions = 30 × 30 cm.**

**[Oct. '97 M.U. April '97 Bharathiyar University]**

### Given

Thickness of plate  $L = 2 \text{ mm} = 0.002 \text{ m}$

Initial temperature  $T_0 = 400^\circ\text{C} + 273 = 673 \text{ K}$

Final temperature  $T = 30^\circ\text{C} + 273 = 303 \text{ K}$

Intermediate temperature  $T = 50^\circ\text{C} + 273 = 323 \text{ K}$

Heat transfer co-efficient  $h = 100 \text{ W/m}^2\text{K}$

Density  $\rho = 8800 \text{ kg/m}^3$

Specific heat  $C_p = 360 \text{ J/kg K}$

Plate dimensions = 30 × 30 cm

### To find

Time required for the plate to reach 50°C.

[From HMT data book Page No.2]

**Solution:**

Thermal conductivity of the copper  $K = 386 \text{ W/mK}$

For slab,

$$\begin{aligned}\text{Characteristic length } L_c &= \frac{L}{2} \\ &= \frac{0.002}{2} \\ \boxed{L_c = 0.001 \text{ m}}\end{aligned}$$

We know,

$$\begin{aligned}\text{Biot number } B_i &= \frac{hL_c}{K} \\ &= \frac{100 \times 0.001}{386} \\ B_i &= 2.59 \times 10^{-4} < 0.1\end{aligned}$$

Biot number value is less than 0.1. So this is lumped heat analysis type problem.

For lumped parameter system,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[ \frac{-hA}{C_p \times V \times \rho} \times t \right]} \dots \dots \dots (1)$$

[From HMT data book Page No.48]

We know,

$$\text{Characteristics length } L_c = \frac{V}{A}$$

$$\begin{aligned}(1) \Rightarrow \frac{T - T_\infty}{T_0 - T_\infty} &= e^{\left[ \frac{-h}{C_p \times L_c \times \rho} \times t \right]} \\ \Rightarrow \frac{323 - 303}{673 - 303} &= e^{\left[ \frac{-100}{360 \times 0.001 \times 8800} \times t \right]} \\ \Rightarrow \boxed{t = 92.43 \text{ s}}\end{aligned}$$

Time required for the plate to reach  $50^\circ\text{C}$  is 92.43 s.

8. A steel ball (specific heat = 0.46 kJ/kgK. and thermal conductivity = 35 W/mK) having 5 cm diameter and initially at a uniform temperature of 450°C is suddenly placed in a control environment in which the temperature is maintained at 100°C. Calculate the time required for the balls to attained a temperature of 150°C. Take  $h = 10\text{W/m}^2\text{K}$ .

[M.U. April-2000, 2001, 2002, Bharathiyar Uni. April 98] Bharathiyar Uni. April 98]

### Given

Specific heat  $C_p = 0.46 \text{ kJ/kg K} = 460 \text{ J/kg K}$

Thermal conductivity  $K = 35 \text{ W/mK}$

Diameter of the sphere  $D = 5 \text{ cm} = 0.05 \text{ m}$

Radius of the sphere  $R = 0.025 \text{ m}$

Initial temperature  $T_0 = 450^\circ\text{C} + 273 = 723 \text{ K}$

Final temperature  $T_\infty = 100^\circ\text{C} + 273 = 373 \text{ K}$

Intermediate temperature  $T = 150^\circ\text{C} + 273 = 423 \text{ K}$

Heat transfer co-efficient  $h = 10 \text{ W/m}^2\text{K}$

### To find

Time required for the ball to reach 150°C

[From HMT data book Page No.1]

### Solution

Density of steel is  $7833 \text{ kg/m}^3$

$$\rho = 7833 \text{ kg/m}^3$$

For sphere,

$$\text{Characteristic Length } L_c = \frac{R}{3}$$

$$= \frac{0.025}{3}$$

$$L_c = 8.33 \times 10^{-3} \text{ m}$$

We know,

$$\begin{aligned} \text{Biot number } B_i &= \frac{hL_c}{K} \\ &= \frac{10 \times 8.3 \times 10^{-3}}{35} \end{aligned}$$

$$B_i = 2.38 \times 10^{-3} < 0.1$$

Biot number value is less than 0.1. So this is lumped heat analysis type problem.

For lumped parameter system,

$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = e^{\left[ \frac{-hA}{C_p \times V \times \rho} \times t \right]} \dots\dots\dots(1)$$

[From HMT data book Page No.48]

We know,

$$\text{Characteristics length } L_c = \frac{V}{A}$$

$$\begin{aligned} (1) \Rightarrow \frac{T - T_{\infty}}{T_0 - T_{\infty}} &= e^{\left[ \frac{-h}{C_p \times L_c \times \rho} \times t \right]} \\ \Rightarrow \frac{423 - 373}{723 - 373} &= e^{\left[ \frac{-10}{460 \times 8.33 \times 10^{-3} \times 7833} \times t \right]} \\ \Rightarrow \ln \frac{423 - 373}{723 - 373} &= \frac{-10}{460 \times 8.33 \times 10^{-3} \times 7833} \times t \\ \Rightarrow \boxed{t = 5840.54 \text{ s}} \end{aligned}$$

Time required for the ball to reach 150°C is 5840.54 s.

**9. Alloy steel ball of 2 mm diameter heated to 800°C is quenched in a bath at 100°C. The material properties of the ball are  $K = 205 \text{ kJ/m hr K}$ ,  $\rho = 7860 \text{ kg/m}^3$ ,  $C_p = 0.45 \text{ kJ/kg K}$ ,  $h = 150 \text{ KJ/ hr m}^2 \text{ K}$ . Determine (i) Temperature of ball after 10 second and (ii) Time for ball to cool to 400°C.**

**Given**

Diameter of the ball  $D = 12 \text{ mm} = 0.012 \text{ m}$

Radius of the ball  $R = 0.006 \text{ m}$

Initial temperature  $T_0 = 800^\circ\text{C} + 273 = 1073 \text{ K}$

Final temperature  $T_{\infty} = 100^\circ\text{C} + 273 = 373 \text{ K}$

Thermal conductivity  $K = 205 \text{ kJ/m hr K}$

$$\begin{aligned} &= \frac{205 \times 1000 \text{ J}}{3600 \text{ s mK}} \\ &= 56.94 \text{ W/mK} \quad [\because \text{J/s} = \text{W}] \end{aligned}$$

Density  $\rho = 7860 \text{ kg/m}^3$

Specific heat  $C_p = 0.45 \text{ kJ/kg K}$   
 $= 450 \text{ J/kg K}$

Heat transfer co-efficient  $h = 150 \text{ kJ/hr m}^2 \text{ K}$

$$\begin{aligned} &= \frac{150 \times 1000 \text{ J}}{3600 \text{ s m}^2 \text{ K}} \\ &= 41.66 \text{ W/m}^2 \text{ K} \end{aligned}$$

**Solution**

**Case (i) Temperature of ball after 10 sec.**

For sphere,

$$\begin{aligned}\text{Characteristic Length } L_c &= \frac{R}{3} \\ &= \frac{0.006}{3}\end{aligned}$$

$$L_c = 0.002 \text{ m}$$

We know,

$$\begin{aligned}\text{Biot number } B_i &= \frac{hL_c}{K} \\ &= \frac{41.667 \times 0.002}{56.94}\end{aligned}$$

$$B_i = 1.46 \times 10^{-3} < 0.1$$

Biot number value is less than 0.1. So this is lumped heat analysis type problem.

For lumped parameter system,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[ \frac{-hA}{C_p \times V \times \rho} \times t \right]} \dots\dots\dots (1)$$

[From HMT data book Page No.48]

We know,

$$\text{Characteristics length } L_c = \frac{V}{A}$$

$$(1) \Rightarrow \frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[ \frac{-h}{C_p \times L_c \times \rho} \times t \right]} \dots\dots\dots (2)$$

$$\Rightarrow \frac{T - 373}{1073 - 373} = e^{\left[ \frac{-41.667}{450 \times 0.002 \times 7860} \times 10 \right]}$$

$$\Rightarrow \boxed{T = 1032.95 \text{ K}}$$

**Case (ii) Time for ball to cool to 400°C**

$$\therefore T = 400^\circ\text{C} + 273 = 673 \text{ K}$$

$$(2) \Rightarrow \frac{T - T_{\infty}}{T_0 - T_{\infty}} = e^{\left[ \frac{-h}{C_p \times L_c \times \rho} \times t \right]} \dots\dots(2)$$

$$\Rightarrow \frac{673 - 373}{1073 - 373} = e^{\left[ \frac{-41.667}{450 \times 0.002 \times 7860} \times t \right]}$$

$$\Rightarrow \ln \left[ \frac{673 - 373}{1073 - 373} \right] = \frac{-41.667}{450 \times 0.002 \times 7860} \times t$$

$$\Rightarrow \boxed{t = 143.849 \text{ s}}$$

**10. A large steel plate 5 cm thick is initially at a uniform temperature of 400°C. It is suddenly exposed on both sides to a surrounding at 60°C with convective heat transfer co-efficient of 285 W/m<sup>2</sup>K. Calculate the centre line temperature and the temperature inside the plate 1.25 cm from themed plane after 3 minutes.**

**Take K for steel = 42.5 W/mK,  $\alpha$  for steel = 0.043 m<sup>2</sup>/hr.**

**Given**

Thickness  $L = 5 \text{ cm} = 0.05 \text{ m}$

Initial temperature  $T_i = 400^\circ\text{C} + 273 = 673 \text{ K}$

Final temperature  $T_{\infty} = 60^\circ\text{C} + 273 = 333 \text{ K}$

Distance  $x = 1.25 \text{ mm} = 0.0125 \text{ m}$

Time  $t = 3 \text{ minutes} = 180 \text{ s}$

Heat transfer co-efficient  $h = 285 \text{ W/m}^2\text{K}$

Thermal diffusivity  $\alpha = 0.043 \text{ m}^2/\text{hr} = 1.19 \times 10^{-5} \text{ m}^2/\text{s}$ .

Thermal conductivity  $K = 42.5 \text{ W/mK}$ .

**Solution**

**For Plate :**

$$\begin{aligned} \text{Characteristic Length } L_c &= \frac{L}{2} \\ &= \frac{0.05}{2} \end{aligned}$$

$$\boxed{L_c = 0.025 \text{ m}}$$

We know,

$$\begin{aligned} \text{Biot number } B_i &= \frac{hL_c}{K} \\ &= \frac{285 \times 0.025}{42.5} \end{aligned}$$

$$\Rightarrow \boxed{B_i = 0.1675}$$

$0.1 < B_i < 100$ , So this is infinite solid type problem.

**Infinite Solids**

**Case (i)**

[To calculate centre line temperature (or) Mid plane temperature for infinite plate, refer HMT data book Page No.59 Heisler chart].

$$\begin{aligned} \text{X axis} \rightarrow \text{Fourier number} &= \frac{\alpha t}{L_c^2} \\ &= \frac{1.19 \times 10^{-5} \times 180}{(0.025)^2} \end{aligned}$$

$$\boxed{\text{X axis} \rightarrow \text{Fourier number} = 3.42}$$

$$\text{Curve} = \frac{hL_c}{K}$$

$$= \frac{285 \times 0.025}{42.5} = 0.167$$

$$\boxed{\text{Curve} = \frac{hL_c}{K} = 0.167}$$

X axis value is 3.42, curve value is 0.167, corresponding Y axis value is 0.64

$$\text{Y axis} = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.64$$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = 0.64$$

$$\Rightarrow \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.64$$

$$\Rightarrow \frac{T_0 - 333}{673 - 333} = 0.64$$

$$\Rightarrow \boxed{T_0 = 550.6 \text{ K}}$$

$$\boxed{\text{Center line temperature } T_0 = 550.6 \text{ K}}$$

**Case (ii)** Temperature ( $T_x$ ) at a distance of 0.0125 m from mid plane

[Refer HMT data book Page No.60, Heisler chart]

$$\text{X axis} \rightarrow \text{Biot number } B_i = \frac{hL_c}{K} = 0.167$$

$$\text{Curve} \rightarrow \frac{x}{L_c} = \frac{0.0125}{0.025} = 0.5$$

X axis value is 0.167, curve value is 0.5, corresponding Y axis value is 0.97.

$$\frac{T_x - T_\infty}{T_0 - T_\infty} = 0.97$$

$$Y \text{ axis} = \frac{T_x - T_\infty}{T_0 - T_\infty} = 0.97$$

$$\Rightarrow \frac{T_x - T_\infty}{T_0 - T_\infty} = 0.97$$

$$\Rightarrow \frac{T_x - 333}{550.6 - 333} = 0.97$$

$$\Rightarrow \boxed{T_x = 544 \text{ K}}$$

Temperature inside the plate 1.25 cm from the mid plane is 544 K.

## Unit-1 Convection

### Part-A

#### 1. Define convection.

Convection is a process of heat transfer that will occur between a solid surface and a fluid medium when they are at different temperatures.

#### 2. Define Reynolds number (Re) & Prandtl number (Pr).

**Reynolds number** is defined as the ratio of inertia force to viscous force.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

**Prandtl number** is the ratio of the momentum diffusivity of the thermal diffusivity.

$$Pr = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$$

#### 3. Define Nusselt number (Nu).

It is defined as the ratio of the heat flow by convection process under an unit temperature gradient to the heat flow rate by conduction under an unit temperature gradient through a stationary thickness (L) of metre.

$$\text{Nusselt number (Nu)} = \frac{Q_{\text{conv}}}{Q_{\text{cond}}}$$

#### 4. Define Grash of number (Gr) & Stanton number (St).

It is defined as the ratio of product of inertia force and buoyancy force to the square of viscous force.

$$Gr = \frac{\text{Inertia force} \times \text{Buoyancy force}}{(\text{Viscous force})^2}$$

**Stanton number** is the ratio of nusselt number to the product of Reynolds number and prandtl number.

$$St = \frac{Nu}{Re \times Pr}$$

### 5. What is meant by Newtonian and non – Newtonian fluids?

The fluids which obey the Newton's Law of viscosity are called Newtonian fluids and those which do not obey are called non – Newtonian fluids.

### 6. What is meant by laminar flow and turbulent flow?

**Laminar flow:** Laminar flow is sometimes called stream line flow. In this type of flow, the fluid moves in layers and each fluid particle follows a smooth continuous path. The fluid particles in each layer remain in an orderly sequence without mixing with each other.

**Turbulent flow:** In addition to the laminar type of flow, a distinct irregular flow is frequently observed in nature. This type of flow is called turbulent flow. The path of any individual particle is zig – zag and irregular. Fig. shows the instantaneous velocity in laminar and turbulent flow.

### 7. What is meant by free or natural convection & forced convection?

If the fluid motion is produced due to change in density resulting from temperature gradients, the mode of heat transfer is said to be free or natural convection.

If the fluid motion is artificially created by means of an external force like a blower or fan, that type of heat transfer is known as forced convection.

### 8. Define boundary layer thickness.

The thickness of the boundary layer has been defined as the distance from the surface at which the local velocity or temperature reaches 99% of the external velocity or temperature.

### 9. What is the form of equation used to calculate heat transfer for flow through cylindrical pipes?

$$Nu = 0.023 (Re)^{0.8} (Pr)^n$$

$n = 0.4$  for heating of fluids

$n = 0.3$  for cooling of fluids

#### 10. What is meant by Newtonian and non – Newtonian fluids?

The fluids which obey the Newton's Law of viscosity are called Newtonian fluids and those which do not obey are called non – Newtonian fluids.

Part-B

1. Air at 20°C, at a pressure of 1 bar is flowing over a flat plate at a velocity of 3 m/s. if the plate maintained at 60°C, calculate the heat transfer per unit width of the plate. Assuming the length of the plate along the flow of air is 2m.

**Given :** Fluid temperature  $T_{\infty} = 20^{\circ}\text{C}$ , Pressure  $p = 1 \text{ bar}$ ,  
Velocity  $U = 3 \text{ m/s}$ , Plate surface temperature  $T_w = 60^{\circ}\text{C}$ ,  
Width  $W = 1 \text{ m}$ , Length  $L = 2 \text{ m}$ .

**Solution :** We know,

$$\begin{aligned}\text{Film temperature } T_f &= \frac{T_w + T_{\infty}}{2} \\ &= \frac{60 + 20}{2} \\ T_f &= 40^{\circ}\text{C}\end{aligned}$$

**Properties of air at 40°C:**

Density  $\rho = 1.129 \text{ Kg/m}^3$  Thermal conductivity  $K = 26.56 \times 10^{-3} \text{ W/mK}$ ,  
Kinematic viscosity  $\nu = 16.96 \times 10^{-6} \text{ m}^2/\text{s}$ . Prandtl number  $Pr = 0.699$

We know, Reynolds number  $Re = \frac{UL}{\nu} = \frac{3 \times 2}{16.96 \times 10^{-6}} = 35.377 \times 10^4$

$$Re = 35.377 \times 10^4 < 5 \times 10^5$$

Reynolds number value is less than  $5 \times 10^5$ , so this is laminar flow.

For flat plate, Laminar flow,

$$\text{Local Nusselt Number } Nu_x = 0.332 (Re)^{0.5} (Pr)^{0.333}$$

$$Nu_x = 0.332 (35.377 \times 10^4)^{0.5} \times (0.699)^{0.333}$$

$$Nu_x = 175.27$$

We know that,

$$\text{Local Nusselt Number } Nu_x = \frac{h_s \times L}{K}$$

$$\Rightarrow 175.27 = \frac{h_s \times 2}{26.56 \times 10^{-3}}$$

$$\text{Local heat transfer coefficient } h_x = 2.327 \text{ W/m}^2\text{K} \quad \text{We know,}$$

$$\text{Average heat transfer coefficient } h = 2 \times h_x \quad h = 2 \times 2.327$$

$$h = 4.65 \text{ W/m}^2\text{K}$$

$$\text{Heat transfer } Q = h A (T_w - T_\infty)$$

$$= 4.65 \times 2 (60 - 20)$$

$$[\because \text{Area} = \text{width} \times \text{length} = 1 \times 2 = 2]$$

$$Q = 372 \text{ Watts.}$$

**2. Air at 20°C at atmospheric pressure flows over a flat plate at a velocity of 3 m/s. if the plate is 1 m wide and 80°C, calculate the following at x = 300 mm.**

1. Hydrodynamic boundary layer thickness,
2. Thermal boundary layer thickness,
3. Local friction coefficient,
4. Average friction coefficient,
5. Local heat transfer coefficient
6. Average heat transfer coefficient,
7. Heat transfer.

**Given:** Fluid temperature  $T_\infty = 20^\circ\text{C}$

Velocity  $U = 3 \text{ m/s}$

Wide  $W = 1 \text{ m}$

Surface temperature  $T_w = 80^\circ\text{C}$

Distance  $x = 300 \text{ mm} = 0.3 \text{ m}$

**Solution:** We know, Film temperature  $T_f = \frac{T_w + T_\infty}{2}$

$$= \frac{80 + 20}{2}$$

$$T_f = 50^\circ\text{C}$$

Properties of air at  $50^\circ\text{C}$

Density  $\rho = 1.093 \text{ kg/m}^3$

Kinematic viscosity  $\nu = 17.95 \times 10^{-6} \text{ m}^2/\text{s}$

Prandtl number  $Pr = 0.698$

Thermal conductivity  $K = 28.26 \times 10^{-3} \text{ W/mK}$

We know,

$$\text{Reynolds number } Re = \frac{UL}{\nu}$$

$$= \frac{3 \times 0.3}{17.95 \times 10^{-6}}$$

$$Re = 5.01 \times 10^4 < 5 \times 10^5$$

Since  $Re < 5 \times 10^5$ , flow is laminar

For Flat plate, laminar flow,

### 1. Hydrodynamic boundary layer thickness:

$$\begin{aligned}\delta_{hx} &= 5 \times x \times (Re)^{-0.5} \\ &= 5 \times 0.3 \times (5.01 \times 10^4)^{-0.5} \\ \delta_{hx} &= 6.7 \times 10^{-3} \text{ m}\end{aligned}$$

### 2. Thermal boundary layer thickness:

$$\begin{aligned}\delta_{TX} &= \delta_{hx} (Pr)^{-0.333} \\ \Rightarrow \delta_{TX} &= (6.7 \times 10^{-3}) (0.698)^{-0.333} \\ \delta_{TX} &= 7.5 \times 10^{-3} \text{ m}\end{aligned}$$

### 3. Local Friction coefficient:

$$\begin{aligned}C_{fx} &= 0.664 (Re)^{-0.5} \\ &= 0.664 (5.01 \times 10^4)^{-0.5} \\ C_{fx} &= 2.96 \times 10^{-3}\end{aligned}$$

#### 4. Average friction coefficient:

$$\begin{aligned}\overline{C_{fL}} &= 1.328 (\text{Re})^{-0.5} \\ &= 1.328 (5.01 \times 10^4)^{-0.5} \\ &= 5.9 \times 10^{-3} \\ \overline{C_{fL}} &= 5.9 \times 10^{-3}\end{aligned}$$

#### 5. Local heat transfer coefficient ( $h_x$ ):

$$\begin{aligned}\text{Local Nusselt Number } Nu_x &= 0.332 (\text{Re})^{0.5} (\text{Pr})^{0.333} \\ &= 0.332 (5.01 \times 10^4) (0.698)^{0.333} \\ Nu_x &= 65.9\end{aligned}$$

We know

Local Nusselt Number

$$\begin{aligned}Nu_x &= \frac{h_x \times L}{K} \\ 65.9 &= \frac{h_x \times 0.3}{23.26 \times 10^{-3}} \quad [\because x = L = 0.3\text{m}] \\ \Rightarrow h_x &= 6.20 \text{ W/m}^2\text{K}\end{aligned}$$

Local heat transfer coefficient  $h_x = 6.20 \text{ W/m}^2\text{K}$

#### 6. Average heat transfer coefficient ( $h$ ):

$$\begin{aligned}h &= 2 \times h_x \\ &= 2 \times 6.20 \\ h &= 12.41 \text{ W/m}^2\text{K}\end{aligned}$$

#### 7. Heat transfer:

$$Q = h A (T_w - T_\infty)$$

We know that,  $= 12.41 \times (1 \times 0.3) (80 - 20)$

$$Q = 23.38 \text{ Watts}$$

3. Air at 30°C flows over a flat plate at a velocity of 2 m/s. The plate is 2 m long and 1.5 m wide. Calculate the following:

1. Boundary layer thickness at the trailing edge of the plate,
2. Total drag force,
3. Total mass flow rate through the boundary layer between  $x = 40 \text{ cm}$  and  $x = 85 \text{ cm}$ .

**Given:** Fluid temperature  $T_{\infty} = 30^{\circ}\text{C}$

Velocity  $U = 2 \text{ m/s}$

Length  $L = 2 \text{ m}$

Wide  $W = 1.5 \text{ m}$

**To find:**

1. Boundary layer thickness
2. Total drag force.
3. Total mass flow rate through the boundary layer between  $x = 40 \text{ cm}$  and  $x = 85 \text{ cm}$ .

**Solution:** Properties of air at  $30^{\circ}\text{C}$

$$\rho = 1.165 \text{ kg/m}^3$$

$$\nu = 16 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.701$$

$$K = 26.75 \times 10^{-3} \text{ W/mK}$$

We know,

$$\text{Reynolds number } Re = \frac{UL}{\nu}$$

$$= \frac{2 \times 2}{16 \times 10^{-6}}$$

$$Re = 2.5 \times 10^5 < 5 \times 10^5$$

Since  $Re < 5 \times 10^5$ , flow is laminar

For flat plate, laminar flow, [from HMT data book, Page No.99]

Hydrodynamic boundary layer thickness

$$\begin{aligned}\delta_{hx} &= 5 \times x \times (\text{Re})^{-0.5} \\ &= 5 \times 2 \times (2.5 \times 10^5)^{-0.5} \\ \delta_{hx} &= 0.02 \text{ m}\end{aligned}$$

Thermal boundary layer thickness,

$$\begin{aligned}\delta_{tx} \delta_{hx} &\times (\text{Pr})^{-0.333} \\ &= 0.02 \times (0.701)^{-0.333} \\ \delta_{tx} &= 0.0225 \text{ m}\end{aligned}$$

We know,

Average friction coefficient,

$$\begin{aligned}\overline{C_{fL}} &= 1.328 (\text{Re})^{-0.5} \\ &= 1.328 \times (2.5 \times 10^5)^{-0.5} \\ \overline{C_{fL}} &= 2.65 \times 10^{-3}\end{aligned}$$

We know

$$\begin{aligned}\overline{C_{fL}} &= \frac{t}{\frac{\rho U^2}{2}} \\ \Rightarrow 2.65 \times 10^{-3} &= \frac{t}{\frac{1.165 \times (2)^2}{2}}\end{aligned}$$

$$\Rightarrow \text{Average shear stress } t = 6.1 \times 10^{-3} \text{ N/m}^2$$

Drag force = Area  $\times$  Average shear stress

$$= 2 \times 1.5 \times 6.1 \times 10^{-3}$$

$$\text{Drag force} = 0.018 \text{ N}$$

Drag force on two sides of the plate

$$= 0.018 \times 2$$

$$= 0.036 \text{ N}$$

Total mass flow rate between  $x = 40$  cm and  $x = 85$  cm.

$$\Delta m = \frac{5}{8} \rho U [\delta_{hx} = 85 - \delta_{hx} = 40]$$

Hydrodynamic boundary layer thickness

$$\begin{aligned} \delta_{hx=0.85} &= 5 \times x \times (\text{Re})^{-0.5} \\ &= 5 \times 0.85 \times \left[ \frac{U \times x}{\nu} \right]^{-0.5} \end{aligned}$$

$$= 5 \times 0.85 \times \left[ \frac{2 \times 0.85}{16 \times 10^{-6}} \right]^{-0.5}$$

$$\delta_{HX=0.85} = 0.0130 \text{ m}$$

$$\delta_{hx=0.40} = 5 \times x \times (\text{Re})^{-0.5}$$

$$= 5 \times 0.40 \times \left( \frac{U \times x}{\nu} \right)^{-0.5}$$

$$= 5 \times 0.40 \times \left( \frac{2 \times 0.40}{16 \times 10^{-6}} \right)^{-0.5}$$

$$\delta_{HX=0.40} = 8.9 \times 10^{-3} \text{ m}$$

$$(1) \Rightarrow \Delta m = \frac{5}{8} \times 1.165 \times 2 [0.0130 - 8.9 \times 10^{-3}]$$

$$\Delta m = 5.97 \times 10^{-3} \text{ Kg/s,}$$

**4. Air at 290°C flows over a flat plate at a velocity of 6 m/s. The plate is 1m long and 0.5 m wide. The pressure of the air is 6 kN/m<sup>2</sup>. If the plate is maintained at a temperature of 70°C, estimate the rate of heat removed from the plate.**

**Given :** Fluid temperature  $T_{\infty} = 290^{\circ}\text{C}$

Velocity  $U = 6$  m/s. Length  $L = 1$  m

Wide  $W = 0.5$  m

Pressure of air  $P = 6 \text{ kN/m}^2 = 6 \times 10^3 \text{ N/m}^2$

Plate surface temperature  $T_w = 70^{\circ}\text{C}$

**To find:** Heat removed from the plate

**Solution:** We know, Film temperature  $T_f = \frac{T_w + T_{\infty}}{2}$

$$= \frac{70 + 290}{2}$$

$$T_f = 180^\circ\text{C}$$

Properties of air at  $180^\circ\text{C}$  (At atmospheric pressure)

$$\rho = 0.799 \text{ Kg/m}^3$$

$$\nu = 32.49 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.681$$

$$K = 37.80 \times 10^{-3} \text{ W/mK}$$

**Note:** Pressure other than atmospheric pressure is given, so kinematic viscosity will vary with pressure. Pr, K,  $C_p$  are same for all pressures.

$$\text{Kinematic viscosity } \nu = \nu_{\text{atm}} \times \frac{P_{\text{atm}}}{P_{\text{given}}}$$

$$\Rightarrow \nu = 32.49 \times 10^{-6} \frac{1 \text{ bar}}{6 \times 10^3 \text{ N/m}^2}$$

$$[\because \text{Atmospheric pressure} = 1 \text{ bar}]$$

$$= 32.49 \times 10^{-6} \times \frac{10^5 \text{ N/m}^2}{6 \times 10^3 \text{ N/m}^3}$$

$$[\because 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2]$$

$$\text{Kinematic viscosity } \nu = 5.145 \times 10^{-4} \text{ m}^2/\text{s}.$$

$$\text{We know, Reynolds number } \text{Re} = \frac{UL}{\nu}$$

$$= \frac{6 \times 1}{5.145 \times 10^{-4}}$$

$$\text{Re} = 1.10 \times 10^4 - 5 \times 10^5$$

Since  $\text{Re} < 5 \times 10^5$ , flow is laminar

For plate, laminar flow,

Local nusselt number

$$\text{NU}_x = 0.332 (\text{Re})^{0.5} (\text{Pr})^{0.333}$$

$$= 0.332 (1.10 \times 10^4)^{0.5} (0.681)^{0.333}$$

$$\text{NU}_x = 30.63$$

We know  $NU_x = \frac{h_x L}{K}$

$$30.63 = \frac{h_x \times 1}{37.80 \times 10^{-3}} \quad [\because L = 1 \text{ m}]$$

Local heat transfer coefficient  $h_x = 1.15 \text{ W/m}^2\text{K}$

We know

Average heat transfer coefficient  $h = 2 \times h_x$

$$h = 2 \times 1.15$$

$$h = 2.31 \text{ W/m}^2\text{K}$$

We know

$$\begin{aligned} \text{Heat transferred } Q &= h A (T_\infty - T_w) \\ &= 2.31 \times (1 \times 0.5) \times (563 - 343) \end{aligned}$$

$$Q = 254.1 \text{ W}$$

$$\begin{aligned} \text{Heat transfer from both side of the plate} &= 2 \times 254.1 \\ &= 508.2 \text{ W.} \end{aligned}$$

**5. Air at 40°C flows over a flat plate, 0.8 m long at a velocity of 50 m/s. The plate surface is maintained at 300°C. Determine the heat transferred from the entire plate length to air taking into consideration both laminar and turbulent portion of the boundary layer. Also calculate the percentage error if the boundary layer is assumed to be turbulent nature from the very leading edge of the plate.**

**Given :** Fluid temperature  $T_\infty = 40^\circ\text{C}$ , Length  $L = 0.8 \text{ m}$ , Velocity  $U = 50 \text{ m/s}$ , Plate surface temperature  $T_w = 300^\circ\text{C}$

**To find :**

1. Heat transferred for:

- Entire plate is considered as combination of both laminar and turbulent flow.
- Entire plate is considered as turbulent flow.

2. Percentage error.

**Solution:** We know Film temperature  $T_f = \frac{T_w - T_\infty}{2}$

$$= \frac{300 + 40}{2} = 443 \text{ K}$$

$$T_f = 170^\circ\text{C}$$

Properties of air at  $170^\circ\text{C}$ :

$$\rho = 0.790 \text{ Kg/m}^3$$

$$\nu = 31.10 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.6815$$

$$K = 37 \times 10^{-3} \text{ W/mK}$$

We know

$$\begin{aligned} \text{Reynolds number } \text{Re} &= \frac{UL}{\nu} \\ &= \frac{50 \times 0.8}{31.10 \times 10^{-6}} = 1.26 \times 10^6 \end{aligned}$$

$$\text{Re} = 1.26 \times 10^6 > 5 \times 10^5$$

$\text{Re} > 5 \times 10^5$ , so this is turbulent flow

**Case (i):** Laminar – turbulent combined. [It means, flow is laminar upto Reynolds number value is  $5 \times 10^5$ , after that flow is turbulent]

$$\text{Average nusselt number} = \text{Nu} = (\text{Pr})^{0.333} (\text{Re})^{0.8} - 871$$

$$\text{Nu} = (0.6815)^{0.333} [0.037 (1.26 \times 10^6)^{0.8} - 871]$$

$$\text{Average nusselt number } \text{Nu} = 1705.3$$

$$\text{We know } \text{Nu} = \frac{hL}{K}$$

$$1705.3 = \frac{h \times 0.8}{37 \times 10^{-3}}$$

$$h = 78.8 \text{ W/m}^2\text{K}$$

Average heat transfer coefficient

$$h = 78.8 \text{ W/m}^2\text{K}$$

$$\text{Heat transfer } Q_1 = h \times A \times (T_w + T_\infty)$$

$$= h \times L \times W \times (T_w + T_\infty)$$

$$= 78.8 \times 0.8 \times 1 \times (300 - 40)$$

$$Q_1 = 16390.4 \text{ W}$$

**Case (ii) :** Entire plate is turbulent flow:

Local nusselt number}  $Nu_x = 0.0296 \times (Re)^{0.8} \times (Pr)^{0.333}$

$$NU_x = 0.0296 \times (1.26 \times 10^6)^{0.8} \times (0.6815)^{0.333}$$

$$NU_x = 1977.57$$

We know  $NU_x = \frac{h_x \times L}{K}$

$$1977.57 = \frac{h_x \times 0.8}{37 \times 10^{-3}}$$

$$h_x = 91.46 \text{ W/m}^2\text{K}$$

Local heat transfer coefficient  $h_x = 91.46 \text{ W/m}^2\text{K}$

Average heat transfer coefficient (for turbulent flow)

$$h = 1.24 \times h_x$$

$$= 1.24 \times 91.46$$

Average heat transfer coefficient}  $h = 113.41 \text{ W/m}^2\text{K}$

We know Heat transfer  $Q_2 = h \times A \times (T_w + T_\infty)$

$$= h \times L \times W \times (T_w + T_\infty)$$

$$= 113.41 \times 0.8 \times 1 (300 - 40)$$

$$Q_2 = 23589.2 \text{ W}$$

2. Percentage error =  $\frac{Q_2 - Q_1}{Q_1}$

$$= \frac{23589.2 - 16390.4}{16390.4} \times 100$$

$$= 43.9\%$$

**6. 250 Kg/hr of air are cooled from 100°C to 30°C by flowing through a 3.5 cm inner diameter pipe coil bent in to a helix of 0.6 m diameter. Calculate the value of air side heat transfer coefficient if the properties of air at 65°C are**

$$K = 0.0298 \text{ W/mK}$$

$$\mu = 0.003 \text{ Kg/hr} - \text{m}$$

$$\text{Pr} = 0.7$$

$$\rho = 1.044 \text{ Kg/m}^3$$

**Given :** Mass flow rate in = 205 kg/hr

$$= \frac{205}{3600} \text{ Kg/s in} = 0.056 \text{ Kg/s}$$

Inlet temperature of air  $T_{\text{mi}} = 100^\circ\text{C}$

Outlet temperature of air  $T_{\text{mo}} = 30^\circ\text{C}$

Diameter  $D = 3.5 \text{ cm} = 0.035 \text{ m}$

$$\text{Mean temperature } T_m = \frac{T_{\text{mi}} + T_{\text{mo}}}{2} = 65^\circ\text{C}$$

**To find:** Heat transfer coefficient (h)

**Solution:**

$$\text{Reynolds Number } \text{Re} = \frac{UD}{\nu}$$

$$\text{Kinematic viscosity } \nu = \frac{\mu}{\rho}$$

$$\frac{0.003}{3600} \text{ Kg/s} - \text{m} \\ \frac{1.044 \text{ Kg/m}^3}{\nu}$$

$$\nu = 7.98 \times 10^{-7} \text{ m}^2/\text{s}$$

$$\text{Mass flow rate in} = \rho A U$$

$$0.056 = 1.044 \times \frac{\pi}{4} \times D^2 \times U$$

$$0.056 = 1.044 \times \frac{\pi}{4} \times (0.035)^2 \times U$$

$$\Rightarrow U = 55.7 \text{ m/s}$$

$$(1) \Rightarrow \text{Re} = \frac{UD}{\nu}$$

$$= \frac{55.7 \times 0.035}{7.98 \times 10^{-7}}$$

$$\text{Re} = 2.44 \times 10^6$$

Since  $\text{Re} > 2300$ , flow is turbulent

For turbulent flow, general equation is ( $\text{Re} > 10000$ )

$$\text{Nu} = 0.023 \times (\text{Re})^{0.8} \times (\text{Pr})^{0.3}$$

This is cooling process, so  $n = 0.3$

$$\Rightarrow \text{Nu} = 0.023 \times (2.44 \times 10^6)^{0.8} \times (0.7)^{0.3}$$

$$\text{Nu} = 2661.7$$

$$\text{We know that, } \text{Nu} = \frac{hD}{K}$$

$$2661.7 = \frac{h \times 0.035}{0.0298}$$

Heat transfer coefficient  $h = 2266.2 \text{ W/m}^2\text{K}$

**7. In a long annulus (3.125 cm ID and 5 cm OD) the air is heated by maintaining the temperature of the outer surface of inner tube at 50°C. The air enters at 16°C and leaves at 32°C. Its flow rate is 30 m/s. Estimate the heat transfer coefficient between air and the inner tube.**

**Given :** Inner diameter  $D_i = 3.125 \text{ cm} = 0.03125 \text{ m}$

Outer diameter  $D_o = 5 \text{ cm} = 0.05 \text{ m}$

Tube wall temperature  $T_w = 50^\circ\text{C}$

Inner temperature of air  $T_{mi} = 16^\circ\text{C}$

Outer temperature of air  $t_{mo} = 32^\circ\text{C}$

Flow rate  $U = 30 \text{ m/s}$

**To find:** Heat transfer coefficient (h)

**Solution:**

$$\text{Mean temperature } T_m = \frac{T_{mi} + T_{mo}}{2}$$

$$= \frac{16 + 32}{2}$$

$$T_m = 24^\circ\text{C}$$

Properties of air at  $24^\circ\text{C}$ :

$$\rho = 1.614 \text{ Kg/m}^3$$

$$\nu = 15.9 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.707$$

$$K = 26.3 \times 10^{-3} \text{ W/mK}$$

We know,

Hydraulic or equivalent diameter

$$D_h = \frac{4A}{P} = \frac{4 \times \frac{\pi}{4} [D_o^2 - D_i^2]}{\pi [D_o + D_i]}$$

$$= \frac{(D_o + D_i)(D_o - D_i)}{(D_o + D_i)}$$

$$= D_o - D_i$$

$$= 0.05 - 0.03125$$

$$D_h = 0.01875 \text{ m}$$

$$\text{Reynolds number } Re = \frac{UD_h}{\nu}$$

$$= \frac{30 \times 0.01875}{15.9 \times 10^{-6}}$$

$$Re = 35.3 \times 10^6$$

Since  $Re > 2300$ , flow is turbulent

For turbulent flow, general equation is ( $Re > 10000$ )

$$Nu = 0.023 (Re)^{0.8} (Pr)^n$$

This is heating process. So  $n = 0.4$

$$\Rightarrow Nu = 0.023 \times (35.3 \times 10^3)^{0.8} (0.707)^{0.4}$$

$$Nu = 87.19$$

$$\text{We know } Nu = \frac{hD_h}{K}$$

$$\Rightarrow 87.19 = \frac{h \times 0.01875}{26.3 \times 10^{-3}}$$

$$\Rightarrow h = 122.3 \text{ W/m}^2\text{K}$$

**8. Engine oil flows through a 50 mm diameter tube at an average temperature of 147°C. The flow velocity is 80 cm/s. Calculate the average heat transfer coefficient if the tube wall is maintained at a temperature of 200°C and it is 2 m long.**

$$\text{Given : Diameter } D = 50 \text{ mm} = 0.050 \text{ m}$$

$$\text{Average temperature } T_m = 147^\circ\text{C}$$

$$\text{Velocity } U = 80 \text{ cm/s} = 0.80 \text{ m/s}$$

$$\text{Tube wall temperature } T_w = 200^\circ\text{C}$$

$$\text{Length } L = 2 \text{ m}$$

**To find:** Average heat transfer coefficient ( $h$ )

**Solution :** Properties of engine oil at 147°C

$$\rho = 816 \text{ Kg/m}^3$$

$$\nu = 7 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 116$$

$$K = 133.8 \times 10^{-3} \text{ W/mK}$$

We know

$$\text{Reynolds number } \text{Re} = \frac{UD}{\nu}$$

$$= \frac{0.8 \times 0.05}{7 \times 10^{-6}}$$

$$\text{Re} = 5714.2$$

Since  $\text{Re} < 2300$  flow is turbulent

$$\frac{L}{D} = \frac{2}{0.050} = 40$$

$$10 < \frac{L}{D} < 400$$

For turbulent flow, ( $\text{Re} < 10000$ )

$$\text{Nusselt number } \text{Nu} = 0.036 (\text{Re})^{0.8} (\text{Pr})^{0.33} \left( \frac{D}{L} \right)^{0.055}$$

$$\text{Nu} = 0.036 (5714.2)^{0.8} \times (116)^{0.33} \times \left( \frac{0.050}{2} \right)^{0.055}$$

$$\text{Nu} = 142.8$$

$$\text{We know } \text{Nu} = \frac{hD}{K}$$

$$\Rightarrow 142.8 = \frac{h \times 0.050}{133.8 \times 10^{-3}}$$

$$\Rightarrow h = 382.3 \text{ W/m}^2\text{K}$$

**9. A large vertical plate 4 m height is maintained at 606°C and exposed to atmospheric air at 106°C. Calculate the heat transfer is the plate is 10 m wide.**

**Given :**

Vertical plate length (or) Height  $L = 4 \text{ m}$

Wall temperature  $T_w = 606^\circ\text{C}$

Air temperature  $T_\infty = 106^\circ\text{C}$

Wide  $W = 10 \text{ m}$

**To find: Heat transfer (Q)**

**Solution:**

$$\text{Film temperature } T_f = \frac{T_w + T_\infty}{2}$$

$$= \frac{606 + 106}{2}$$

$$T_f = 356^\circ\text{C}$$

Properties of air at  $356^\circ\text{C} = 350^\circ\text{C}$

$$\rho = 0.566 \text{ Kg/m}^3$$

$$\nu = 55.46 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.676$$

$$K = 49.08 \times 10^{-3} \text{ W/mK}$$

$$\text{Coefficient of thermal expansion } \beta = \frac{1}{T_f \text{ in K}}$$

$$= \frac{1}{356 + 273} = \frac{1}{629}$$

$$\beta = 1.58 \times 10^{-3} \text{ K}^{-1}$$

$$\text{Grashof number } \text{Gr} = \frac{g \times \beta \times L^3 \times \Delta T}{\nu^2}$$

$$\Rightarrow \text{Gr} = \frac{9.81 \times 2.4 \times 10^{-3} \times (4)^3 \times (606 - 106)}{(55.46 \times 10^{-6})^2}$$

$$\text{Gr} = 1.61 \times 10^{11}$$

$$\text{Gr Pr} = 1.61 \times 10^{11} \times 0.676$$

$$\text{Gr Pr} = 1.08 \times 10^{11}$$

Since  $\text{Gr Pr} > 10^9$ , flow is turbulent

For turbulent flow,

$$\text{Nusselt number } Nu = 0.10 [\text{Gr Pr}]^{0.333}$$

$$\Rightarrow Nu = 0.10 [1.08 \times 10^{11}]^{0.333}$$

$$Nu = 471.20$$

We know that,

$$\text{Nusselt number } Nu = \frac{hL}{K}$$

$$\Rightarrow 472.20 = \frac{h \times 4}{49.08 \times 10^{-3}}$$

$$\text{Heat transfer coefficient } h = 5.78 \text{ W/m}^2\text{K}$$

$$\text{Heat transfer } Q = h A \Delta T$$

$$= h \times W \times L \times (T_w - T_\infty)$$

$$= 5.78 \times 10 \times 4 \times (606 - 106)$$

$$Q = 115600 \text{ W}$$

$$Q = 115.6 \times 10^3 \text{ W}$$

**10. A thin 100 cm long and 10 cm wide horizontal plate is maintained at a uniform temperature of 150°C in a large tank full of water at 75°C. Estimate the rate of heat to be supplied to the plate to maintain constant plate temperature as heat is dissipated from either side of plate.**

**Given :**

$$\text{Length of horizontal plate } L = 100 \text{ cm} = 1 \text{ m}$$

$$\text{Wide } W = 10 \text{ cm} = 0.10 \text{ m}$$

$$\text{Plate temperature } T_w = 150^\circ\text{C}$$

$$\text{Fluid temperature } T_\infty = 75^\circ\text{C}$$

**To find:** Heat loss (Q) from either side of plate

**Solution:**

$$\text{Film temperature } T_f = \frac{T_w - T_\infty}{2}$$

$$= \frac{150 + 75}{2}$$

$$T_f = 112.5^\circ\text{C}$$

Properties of water at  $112.5^\circ\text{C}$

$$\rho = 951 \text{ Kg/m}^3$$

$$\nu = 0.264 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 1.55$$

$$K = 683 \times 10^{-3} \text{ W/mK}$$

$$\text{Coefficient of thermal expansion } \beta = \frac{1}{T_f \text{ in K}} = \frac{1}{112.5 + 273}$$

$$\beta = 2.59 \times 10^{-3} \text{ K}^{-1}$$

$$\text{Grashof Number } Gr = \frac{g \times \beta \times L^3 \times \Delta T}{\nu^2}$$

For horizontal plate,

$$\text{Characteristic length } L_c = \frac{W}{2} = \frac{0.10}{2}$$

$$L_c = 0.05 \text{ m}$$

$$(1) \Rightarrow Gr = \frac{9.81 \times 2.59 \times 10^{-3} \times (0.05)^3 \times (150 - 75)}{(0.264 \times 10^{-6})^2}$$

$$Gr = 3.41 \times 10^9$$

$$Gr \text{ Pr} = 3.41 \times 10^9 \times 1.55$$

$$Gr \text{ Pr} = 5.29 \times 10^9$$

Gr Pr value is in between  $8 \times 10^6$  and  $10^{11}$

i.e.,  $8 \times 10^6 < Gr \text{ Pr} < 10^{11}$

**For horizontal plate, upper surface heated:**

$$\text{Nusselt number } Nu = 0.15 (Gr Pr)^{0.333}$$

$$\Rightarrow Nu = 0.15 [5.29 \times 10^9]^{0.333+}$$

$$\Rightarrow Nu = 259.41$$

We know that,

$$\text{Nusselt number } Nu = \frac{h_u L_c}{K}$$

$$259.41 = \frac{h_u \times 0.05}{683 \times 10^{-3}}$$

$$h_u = 3543.6 \text{ W/m}^2\text{K}$$

Upper surface heated, heat transfer coefficient  $h_u = 3543.6 \text{ W/m}^2\text{K}$

**For horizontal plate, lower surface heated:**

$$\text{Nusselt number } Nu = 0.27 [Gr Pr]^{0.25}$$

$$\Rightarrow Nu = 0.27 [5.29 \times 10^9]^{0.25}$$

$$Nu = 72.8$$

We know that,

$$\text{Nusselt number } Nu = \frac{h_l L_c}{K}$$

$$72.8 = \frac{h_l L_c}{K}$$

$$72.8 = \frac{h_l \times 0.05}{683 \times 10^{-3}}$$

$$h_l = 994.6 \text{ W/m}^2\text{K}$$

Lower surface heated, heat transfer coefficient  $h_l = 994.6 \text{ W/m}^2\text{K}$

$$\text{Total heat transfer } Q = (h_u + h_1) \times A \times \Delta T$$

$$= (h_u + h_1) \times W \times L \times (T_w - T_\infty)$$

$$= (3543.6 + 994.6) \times 0.10 \times (150 - 75)$$

$$Q = 34036.5 \text{ W}$$

### Unit-3

#### 1. What is meant by Boiling and condensation?

The change of phase from liquid to vapour state is known as boiling.

The change of phase from vapour to liquid state is known as condensation.

#### 2. Give the applications of boiling and condensation.

Boiling and condensation process finds wide applications as mentioned below.

1. Thermal and nuclear power plant.
2. Refrigerating systems
3. Process of heating and cooling
4. Air conditioning systems

#### 3. What is meant by pool boiling?

If heat is added to a liquid from a submerged solid surface, the boiling process referred to as pool boiling. In this case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment.

#### 4. What is meant by Film wise and Drop wise condensation?

The liquid condensate wets the solid surface, spreads out and forms a continuous film over the entire surface is known as film wise condensation.

In drop wise condensation the vapour condenses into small liquid droplets of various sizes which fall down the surface in a random fashion.

#### 5. Give the merits of drop wise condensation?

In drop wise condensation, a large portion of the area of the plate is directly exposed to vapour. The heat transfer rate in drop wise condensation is 10 times higher than in film condensation.

#### 6. What is heat exchanger?

A heat exchanger is defined as an equipment which transfers the heat from a hot fluid to a cold fluid.

## **7. What are the types of heat exchangers?**

The types of heat exchangers are as follows

1. Direct contact heat exchangers
2. Indirect contact heat exchangers
3. Surface heat exchangers
4. Parallel flow heat exchangers
5. Counter flow heat exchangers
6. Cross flow heat exchangers
7. Shell and tube heat exchangers
8. Compact heat exchangers.

## **8. What is meant by Direct heat exchanger (or) open heat exchanger?**

In direct contact heat exchanger, the heat exchange takes place by direct mixing of hot and cold fluids.

## **9. What is meant by Indirect contact heat exchanger?**

In this type of heat exchangers, the transfer of heat between two fluids could be carried out by transmission through a wall which separates the two fluids.

## **10. What is meant by Regenerators?**

In this type of heat exchangers, hot and cold fluids flow alternately through the same space. Examples: IC engines, gas turbines.

## **11. What is meant by Recuperater (or) surface heat exchangers?**

This is the most common type of heat exchangers in which the hot and cold fluid do not come into direct contact with each other but are separated by a tube wall or a surface.

## **12. What is meant by parallel flow and counter flow heat exchanger?**

In this type of heat exchanger, hot and cold fluids move in the same direction.

In this type of heat exchanger hot and cold fluids move in parallel but opposite directions.

## **13. What is meant by shell and tube heat exchanger?**

In this type of heat exchanger, one of the fluids move through a bundle of tubes enclosed by a shell. The other fluid is forced through the shell and it moves over the outside surface of the tubes.

#### 14. What is meant by compact heat exchangers?

There are many special purpose heat exchangers called compact heat exchangers. They are generally employed when convective heat transfer coefficient associated with one of the fluids is much smaller than that associated with the other fluid.

#### 15. What is meant by LMTD?

We know that the temperature difference between the hot and cold fluids in the heat exchanger varies from point in addition various modes of heat transfer are involved. Therefore based on concept of appropriate mean temperature difference, also called logarithmic mean temperature difference, the total heat transfer rate in the heat exchanger is expressed as

$$Q = U A (\Delta T)_m \text{ Where } U - \text{Overall heat transfer coefficient } W/m^2K \quad A - \text{Area } m^2$$

$(\Delta T)_m$  – Logarithmic mean temperature difference.

#### 16. What is meant by Fouling factor?

We know the surfaces of a heat exchangers do not remain clean after it has been in use for some time. The surfaces become fouled with scaling or deposits. The effect of these deposits the value of overall heat transfer coefficient. This effect is taken care of by introducing an additional thermal resistance called the fouling resistance.

#### 17. What is meant by effectiveness?

The heat exchanger effectiveness is defined as the ratio of actual heat transfer to the maximum possible heat transfer.

$$\text{Effectiveness } \varepsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} = \frac{Q}{Q_{\max}}$$

Part-B

1. Water is boiled at the rate of 24 kg/h in a polished copper pan, 300 mm in diameter, at atmospheric pressure. Assuming nucleate boiling conditions calculate the temperature of the bottom surface of the pan.

**Given :**

$$m = 24 \text{ kg / h}$$

$$= \frac{24 \text{ kg}}{3600 \text{ s}}$$

$$\boxed{m = 6.6 \times 10^{-3} \text{ kg/s}}$$

$$d = 300 \text{ mm} = .3\text{m}$$

### Solution:

We know saturation temperature of water is  $100^\circ\text{C}$

$$\text{i.e. } T_{\text{sat}} = 100^\circ\text{C}$$

Properties of water at  $100^\circ\text{C}$

From HMT data book Page No.13

$$\text{Density } \rho = 961 \text{ kg/m}^3$$

$$\text{Kinematic viscosity } \nu = 0.293 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Prandtl number } Pr = 1.740$$

$$\text{Specific heat } C_{pl} = 4.216 \text{ kJ/kg K} = 4216 \text{ J/kg K}$$

$$\text{Dynamic viscosity } \mu = \rho \nu$$

$$= 961 \times 0.293 \times 10^{-6}$$

$$\boxed{\mu = 281.57 \times 10^{-6} \text{ Ns/m}^2}$$

From steam table (R.S. Khumi Steam table Page No.4)

At  $100^\circ\text{C}$

$$\text{Enthalpy of evaporation } h_{fg} = 2256.9 \text{ kJ/kg}$$

$$\boxed{h_{fg} = 2256.9 \times 10^3 \text{ J/kg}}$$

Specific volume of vapour

$$V_g = 1.673 \text{ m}^3/\text{kg}$$

Density of vapour

$$\rho_v = \frac{1}{v_g}$$

$$\frac{1}{1.673}$$

$$\rho_v = 0.597 \text{ kg/m}^3$$

For nucleate boiling

$$\text{Heat flux } \frac{Q}{A} = \mu_l \times h_{fg} \left| \frac{g \times (\rho_l - \rho_v)}{\sigma} \right| \times \left| \frac{C_{pl} \times \Delta T}{C_{sf} \times h_{fg} P_r^{1.7}} \right|^3$$

We know transferred  $Q = m \times h_{fg}$

Heat transferred  $Q = m \times h_{fg}$ .

$$\frac{Q}{A} = \frac{m h_g}{A}$$

$$\begin{aligned} \frac{Q}{A} &= \frac{6.6 \times 10^{-3} \times 2256.9 \times 10^3}{\frac{\pi}{4} d^2} \\ &= \frac{6.6 \times 10^{-3} \times 2256.9 \times 10^3}{\frac{\pi}{4} (.3)^2} \end{aligned}$$

$$\frac{Q}{A} = 210 \times 10^3 \text{ w/m}^2$$

$\sigma$  = surface tension for liquid vapour interface

At 100°C (From HMT data book Page No.147)

$$\sigma = 58.8 \times 10^{-3} \text{ N/m}$$

For water – copper –  $C_{sf}$  = Surface fluid constant = 013

$$C_{sf} = .013 \quad (\text{From HMT data book Page No.145})$$

Substitute,  $\mu_l$ ,  $h_{fg}$ ,  $\rho_l$ ,  $\rho_v$ ,  $\sigma$ ,  $C_{pl}$ ,  $h_{fg}$ ,  $\frac{Q}{A}$  and  $P_r$  values in Equation (1)

$$(1) \Rightarrow 210 \times 10^3 = 281.57 \times 10^{-6} \times 2256.9 \times 10^3$$

$$\left| \frac{9.81 \times 961 - 597}{58.8 \times 10^{-3}} \right|^{0.5}$$

$$\left| \frac{4216 \times \Delta T}{.013 \times 2256.9 \times 10^3 \times (1.74)^{1.7}} \right|^3$$

$$\Rightarrow \left| \frac{4216 \times \Delta T}{75229.7} \right| = 0.825$$

$$\Rightarrow \Delta T (.56)^3 = .825$$

$$\Rightarrow \Delta T \times .056 = 0.937$$

$$\Delta T = 16.7$$

We know that

$$\text{Excess temperature } \Delta T = T_w - T_{\text{sat}}$$

$$16.7 = T_w - 100^\circ\text{C}.$$

$$T_w = 116.7^\circ\text{C}$$

**2. A nickel wire carrying electric current of 1.5 mm diameter and 50 cm long, is submerged in a water bath which is open to atmospheric pressure. Calculate the voltage at the burn out point, if at this point the wire carries a current of 200A.**

**Given :**

$$D = 1.5\text{mm} = 1.5 \times 10^{-3} \text{ m}; L = 50 \text{ cm} = 0.50\text{m}; \text{Current } I = 200\text{A}$$

**Solution**

We know saturation temperature of water is  $100^\circ\text{C}$

$$\text{i.e. } T_{\text{sat}} = 100^\circ\text{C}$$

Properties of water at  $100^\circ\text{C}$

(From HMT data book Page No.11)

$$\rho l = 961 \text{ kg/m}^3$$

$$v = 0.293 \times 10^{-6} \text{ m}^2/\text{s}$$

$$P_r = 1.740$$

$$C_{pl} = 4.216 \text{ kJ/kg K} = 4216 \text{ J/kg K}$$

$$\mu l = \rho l \times v = 961 \times 0.293 \times 10^{-6}$$

$$\mu l = 281.57 \times 10^{-6} \text{ Ns/m}^2$$

From steam Table at 100°C

R.S. Khurmi Steam table Page No.4

$$h_{fg} = 2256.9 \text{ kJ/kg}$$

$$h_{fg} = 2256.9 \times 10^3 \text{ J/kg}$$

$$v_g = 1.673 \text{ m}^3/\text{kg}$$

$$\rho v = \frac{1}{v_g} = \frac{1}{1.673} = 0.597 \text{ kg/m}^3$$

$\sigma$  = Surface tension for liquid – vapour interface

At 100°C

$$\sigma = 58.8 \times 10^{-3} \text{ N/m (From HMT data book Page No.147)}$$

For nucleate pool boiling critical heat flux (AT burn out)

$$\frac{Q}{A} = 0.18 \times h_{fg} \times \rho v \left[ \frac{\sigma \times g \times (\rho l - \rho v)^{0.25}}{\rho v^2} \right] \text{-----1}$$

(From HMT data book Page No.142)

Substitute  $h_{fg}$ ,  $\rho l$ ,  $\rho v$ ,  $\sigma$  values in Equation (1)

$$(1) \Rightarrow \frac{Q}{A} = 0.18 \times 2256.9 \times 10^3 \times 0.597$$

$$\left[ \frac{58.8 \times 10^{-3} \times 9.81 (961 - .597)}{.597^2} \right]^{0.25}$$

$$\boxed{\frac{Q}{A} = 1.52 \times 10^6 \text{ W/m}^2}$$

We know

Heat transferred  $Q = V \times I$

$$\frac{Q}{A} = \frac{V \times I}{A}$$

$$1.52 \times 10^6 = \frac{V \times 200}{\pi dL} \quad \therefore A = \pi dL$$

$$1.52 \times 10^6 = \frac{V \times 200}{\pi \times 1.5 \times 10^{-3} \times .50}$$

$$\boxed{V = 17.9 \text{ volts}}$$

**3. Water is boiling on a horizontal tube whose wall temperature is maintained at 15°C above the saturation temperature of water. Calculate the nucleate boiling heat transfer coefficient. Assume the water to be at a pressure of 20 atm. And also find the change in value of heat transfer coefficient when**

- 1. The temperature difference is increased to 30°C at a pressure of 10 atm.**
- 2. The pressure is raised to 20 atm at  $\Delta T = 15^\circ\text{C}$**

**Given :**

Wall temperature is maintained at 15°C above the saturation temperature.

$$T_w = 115^\circ\text{C}. \quad \therefore T_{\text{sat}} = 100^\circ\text{C} \quad T_w = 100 + 15 = 115^\circ\text{C}$$

$$p = 10 \text{ atm} = 10 \text{ bar}$$

case (i)

$$\Delta T = 30^\circ\text{C}; p = 10 \text{ atm} = 10 \text{ bar}$$

case (ii)

$p = 20 \text{ atm} = 20 \text{ bar}$ ;  $\Delta T = 15^\circ\text{C}$

**Solution:**

We know that for horizontal surface, heat transfer coefficient

$$h = 5.56 (\Delta T)^3 \quad \text{From HMT data book Page No.128}$$

$$\begin{aligned} h &= 5.56 (T_w - T_{\text{sat}})^3 \\ &= 5.56 (115 - 100)^3 \end{aligned}$$

$$h = 18765 \text{ W/m}^2\text{K}$$

Heat transfer coefficient other than atmospheric pressure

$$\begin{aligned} h_p &= h p^{0.4} \quad \text{From HMT data book Page No.144} \\ &= 18765 \times 10^{0.4} \end{aligned}$$

$$\text{Heat transfer coefficient } h_p = 47.13 \times 10^3 \text{ W/m}^2\text{K}$$

**Case (i)**

$P = 100 \text{ bar}$   $\Delta T = 30^\circ\text{C}$  From HMT data book Page No.144

Heat transfer coefficient

$$h = 5.56 (\Delta T)^3 = 5.56(30)^3$$

$$h = 150 \times 10^3 \text{ W/m}^2\text{K}$$

Heat transfer coefficient other than atmospheric pressure

$$h_p = h_p^{0.4}$$

$$= 150 \times 10^3 (10)^{0.4}$$

$$h_p = 377 \times 10^3 \text{ W/m}^2\text{K}$$

**Case (ii)**

$P = 20 \text{ bar}$ ;  $\Delta T = 15^\circ\text{C}$

Heat transfer coefficient  $h = 5.56 (\Delta T)^3 = 5.56 (15)^3$

$$h = 18765 \text{ W/m}^2\text{K}$$

Heat transfer coefficient other than atmospheric pressure

$$h_p = h_p^{0.4}$$

$$= 18765 (20)^{0.4}$$

$$h_p = 62.19 \times 10^3 \text{ W/m}^2\text{K}$$

**4. A vertical flat plate in the form of fin is 500m in height and is exposed to steam at atmospheric pressure. If surface of the plate is maintained at  $60^\circ\text{C}$ . calculate the following.**

1. The film thickness at the trailing edge
2. Overall heat transfer coefficient
3. Heat transfer rate
4. The condensate mass flow rate.

Assume laminar flow conditions and unit width of the plate.

**Given :**

Height or length  $L = 500 \text{ mm} = 5 \text{ m}$

Surface temperature  $T^w = 60^\circ\text{C}$

**Solution**

We know saturation temperature of water is  $100^\circ\text{C}$

i.e.  $T_{\text{sat}} = 100^\circ\text{C}$

(From R.S. Khurmi steam table Page No.4)

$$h_{fg} = 2256.9 \text{ kJ/kg}$$

$$h_{fg} = 2256.9 \times 10^3 \text{ J/kg}$$

We know

$$\text{Film temperature } T_f = \frac{T_w + T_{\text{sat}}}{2}$$

$$= \frac{60 + 100}{2}$$

$$\boxed{T_f = 80^\circ\text{C}}$$

Properties of saturated water at  $80^\circ\text{C}$

(From HMT data book Page No.13)

$$\rho = 974 \text{ kg/m}^3$$

$$\nu = 0.364 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 668.7 \times 10^{-3} \text{ W/mK}$$

$$\mu = \rho \times \nu = 974 \times 0.364 \times 10^{-6}$$

$$\mu = 354.53 \times 10^{-6} \text{Ns/m}^2$$

1. Film thickness  $\delta_x$

We know for vertical plate

Film thickness

$$\delta_x = \left( \frac{4\mu K \times x \times (T_{\text{sat}} - T_w)}{g \times h_{fg} \times \rho^2} \right)^{0.25}$$

Where

$$X = L = 0.5 \text{ m}$$

$$\delta_x = \frac{4 \times 354.53 \times 10^{-6} \times 668.7 \times 10^{-3} \times 0.5 \times 100 - 60}{9.81 \times 2256.9 \times 10^3 \times 974^2}$$

$$\delta_x = 1.73 \times 10^{-4} \text{ m}$$

2. Average heat transfer coefficient (h)

For vertical surface Laminar flow

$$h = 0.943 \left[ \frac{k_3 \times \rho^2 \times g \times h_{fg}}{\mu \times L \times T_{\text{sat}} - T_w} \right]^{0.25}$$

The factor 0.943 may be replaced by 1.13 for more accurate result as suggested by Mc Adams

$$1.13 \left( \frac{(668.7 \times 10^{-3})^3 \times (974)^2 \times 9.81 \times 2256.9 \times 10^3}{354.53 \times 10^{-6} \times 1.5 \times 100 - 60} \right)^{0.25}$$

$$h = 6164.3 \text{ W/m}^2\text{k}.$$

### 3. Heat transfer rate Q

We know

$$\begin{aligned} Q &= hA(T_{\text{sat}} - T_w) \\ &= h \times L \times W \times (T_{\text{sat}} - T_w) \\ &= 6164.3 \times 0.5 \times 1 \times 100 - 60 \end{aligned}$$

$$Q = 123286 \text{ W}$$

### 4. Condensate mass flow rate m

We know

$$\begin{aligned} Q &= m \times h_{fg} \\ m &= \frac{Q}{h_{fg}} \\ m &= \frac{123286}{2256.9 \times 10^3} \\ m &= 0.054 \text{ kg/s} \end{aligned}$$

**10. Steam at 0.080 bar is arranged to condense over a 50 cm square vertical plate. The surface temperature is maintained at 20°C. Calculate the following.**

- Film thickness at a distance of 25 cm from the top of the plate.**
- Local heat transfer coefficient at a distance of 25 cm from the top of the plate.**
- Average heat transfer coefficient.**
- Total heat transfer**
- Total steam condensation rate.**
- What would be the heat transfer coefficient if the plate is inclined at 30° with horizontal plane.**

**Given :**

Pressure  $P = 0.080 \text{ bar}$

$$\text{Area } A = 50 \text{ cm} \times 50 \text{ cm} = 50 \times 0.50 = 0.25 \text{ m}^2$$

$$\text{Surface temperature } T_w = 20^\circ\text{C}$$

$$\text{Distance } x = 25 \text{ cm} = 0.25 \text{ m}$$

### Solution

Properties of steam at 0.080 bar

(From R.S. Khurmi steam table Page no.7)

$$T_{\text{satj/kg}} = 41.53^\circ\text{C}$$

$$h_{\text{fg}} = 2403.2 \text{ kJ/kg} = 2403.2 \times 10^3 \text{ J/kg}$$

We know

$$\text{Film temperature } T_f = \frac{T_w + T_{\text{sat}}}{2}$$

$$= \frac{20 + 41.53}{2}$$

$$T_f = 30.76^\circ\text{C}$$

Properties of saturated water at  $30.76^\circ\text{C} = 30^\circ\text{C}$

From HMT data book Page No.13

$$\rho = 997 \text{ kg/m}^3$$

$$\nu = 0.83 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 612 \times 10^{-3} \text{ W/mK}$$

$$\mu = \rho \times \nu = 997 \times 0.83 \times 10^{-6}$$

$$\mu = 827.51 \times 10^{-6} \text{ Ns/m}^2$$

a. Film thickness

We know for vertical surfaces

$$\delta x = \left( \frac{4\mu K \times x \times (T_{\text{sat}} - T_w)}{g \times h_{fg} \times \rho^2} \right)^{0.25}$$

(From HMT data book Page No.150)

$$\delta_x = \frac{4 \times 827.51 \times 10^{-6} \times 612 \times 10^{-3} \times .25 \times (41.53 - 20) 100}{9.81 \times 2403.2 \times 10^3 \times 997^2}$$

$$\delta_x = 1.40 \times 10^{-4} \text{ m}$$

b. Local heat transfer coefficient  $h_x$  Assuming Laminar flow

$$h_x = \frac{k}{\delta x}$$

$$h_x = \frac{612 \times 10^{-3}}{1.46 \times 10^{-4}}$$

$$h_x = 4,191 \text{ W/m}^2\text{K}$$

c. Average heat transfer coefficient  $h$

(Assuming laminar flow)

$$h = 0.943 \left[ \frac{k^3 \times \rho^2 \times g \times h_{fg}}{\mu \times L \times T_{\text{sat}} - T_w} \right]^{0.25}$$

The factor 0.943 may be replaced by 1.13 for more accurate result as suggested by Mc Adams

$$h = 0.943 \left[ \frac{k^3 \rho^2 g h_{fg}}{\mu \times L \times T_{\text{sat}} - T_w} \right]^{0.25}$$

Where  $L = 50 \text{ cm} = .5 \text{ m}$

$$h = 1.13 \left| \frac{(612 \times 10^{-3})^3 \times (997)^2 \times 9.81 \times 2403.2 \times 10^3}{827.51 \times 10^{-6} \times .5 \times 41.53 - 20} \right|^{0.25}$$

$$h = 5599.6 \text{ W/m}^2\text{k}$$

d. Heat transfer (Q)

We know

$$Q = hA(T_{\text{sat}} - T_w)$$

$$h \times A \times (T_{\text{sat}} - T_w) \\ = 5599.6 \times 0.25 \times (41.53 - 20)$$

$$Q = 30.139.8 \text{ W}$$

e. Total steam condensation rate (m)

We know

Heat transfer

$$Q = m \times h_{\text{fg}}$$

$$m = \frac{Q}{h_{\text{fg}}}$$

$$m = \frac{30.139.8}{2403.2 \times 10^3}$$

$$m = 0.0125 \text{ kg/s}$$

f. If the plate is inclined at  $\theta$  with horizontal

$$h_{\text{inclined}} = h_{\text{vertical}} \times \sin \theta^{1/4}$$

$$h_{\text{inclined}} = h_{\text{vertical}} \times (\sin 30)^{1/4}$$

$$h_{\text{inclined}} = 5599.6 \times \left(\frac{1}{2}\right)^{1/4}$$

$$h_{\text{inclined}} = 4.708.6 \text{ W/m}^2\text{k}$$

Let us check the assumption of laminar film condensation

We know

$$\text{Reynolds Number } R_e = \frac{4m}{w\mu}$$

where

$W$  = width of the plate = 50cm = .50m

$$R_e = \frac{4 \times .0125}{0.50 \times 827.51 \times 10^{-6}}$$

$$R_e = 120.8 < 1800$$

So our assumption laminar flow is correct.

**5. A condenser is to designed to condense 600 kg/h of dry saturated steam at a pressure of 0.12 bar. A square array of 400 tubes, each of 8 mm diameter is to be used. The tube surface is maintained at 30°C. Calculate the heat transfer coefficient and the length of each tube.**

**Given :**

$$m = 600 \text{ kg/h} = \frac{600}{3600} \text{ kg/s} = 0.166 \text{ kg/s}$$

$$m = 0.166 \text{ kg/s}$$

Pressure  $P = 0.12 \text{ bar}$     No. of tubes = 400

$$\text{Diameter } D = 8\text{mm} = 8 \times 10^{-3}\text{m}$$

$$\text{Surface temperature } T_w = 30^\circ\text{C}$$

**Solution**

Properties of steam at 0.12 bar

From R.S. Khurmi steam table Page No.7

$$T_{\text{sat}} = 49.45^{\circ}\text{C}$$

$$h_{\text{fg}} = 2384.3 \text{ kJ/kg}$$

$$h_{\text{fg}} = 2384.9 \times 10^3 \text{ J/kg}$$

We know

$$\text{Film temperature } T_f = \frac{T_w + T_{\text{sat}}}{2}$$

$$= \frac{30 + 49.45}{2}$$

$$T_f = 39.72^{\circ}\text{C} = 40^{\circ}\text{C}$$

Properties of saturated water at  $40^{\circ}\text{C}$

From HMT data book Page No.13

$$\rho = 995 \text{ kg/m}^3$$

$$\nu = .657 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 628.7 \times 10^{-3} \text{ W/mK}$$

$$\mu = \rho \times \nu = 995 \times 0.657 \times 10^{-6}$$

$$\mu = 653.7 \times 10^{-6} \text{ Ns/m}^2$$

with 400 tubes a  $20 \times 20$  tube of square array could be formed

$$\text{i.e. } N = \sqrt{400} = 20$$

$$N = 20$$

For horizontal bank of tubes heat transfer coefficient.

$$h = 0.728 \left[ \frac{K^3 \rho^2 g h_{\text{fg}}}{\mu D (T_{\text{sat}} - T_w)} \right]^{0.25}$$

From HMT data book Page No.150

$$h = 0.728 \left[ \frac{(628 \times 10^{-3})^3 \times (995)^2 \times 9.81 \times 2384.3 \times 10^3}{653.7 \times 10^{-6} \times 20 \times 8 \times 10^{-3} \times (49.45 - 30)} \right]^{0.25}$$

$$h = 5304.75 \text{ W/m}^2\text{K}$$

We know

Heat transfer

$$Q = hA(T_{\text{sat}} - T_w)$$

No. of tubes = 400

$$Q = 400 \times h \times \pi \times D \times L \times (T_{\text{sat}} - T_w)$$

$$Q = 400 \times 5304.75 \times \pi \times 8 \times 10^{-3} \times 1 (49.45 - 30)$$

$$Q = 1.05 \times 10^6 \times L \dots\dots 1$$

We know

$$Q = m \times h_{\text{fg}}$$

$$= 0.166 \times 2384.3 \times 103$$

$$Q = 0.3957 \times 10^6 \text{ W}$$

$$= 0.3957 \times 10^6 = 1.05 \times 10^6 L$$

$$L = 0.37 \text{ m}$$

Problems on Parallel flow and Counter flow heat exchangers

From HMT data book Page No.135

Formulae used

1. Heat transfer  $Q = UA (\Delta T)_m$

Where

U – Overall heat transfer coefficient, W/m<sup>2</sup>K

A – Area, m<sup>2</sup>

$(\Delta T)_m$  – Logarithmic Mean Temperature Difference. LMTD

For parallel flow

$$(\Delta T)_m = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln \left[ \frac{T_1 - t_1}{T_2 - t_2} \right]}$$

In Counter flow

$$(\Delta T)_m = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln \left[ \frac{T_1 - t_1}{T_2 - t_2} \right]}$$

Where

$T_1$  – Entry temperature of hot fluid °C

$T_2$  – Exit temperature of hot fluid °C

$T_1$  – Entry temperature of cold fluid °C

$T_2$  – Exit temperature of cold fluid °C

## 2. Heat lost by hot fluid = Heat gained by cold fluid

$$Q_h = Q_c$$

$$m_h C_{ph} (T_1 - T_2) = m_c C_{pc} (t_2 - t_1)$$

$M_h$  – Mass flow rate of hot fluid, kg/s

$M_c$  – Mass flow rate of cold fluid kg/s

$C_{ph}$  – Specific heat of hot fluid J/kg K

$C_{pc}$  – Specific heat of cold fluid J/kg L

## 3. Surface area of tube

$$A = \pi D_1 L$$

Where  $D_1$  Inner dia

$$4. Q = m \times h_{fg}$$

Where  $h_{fg}$  – Enthalpy of evaporation J/kg K

## 5. Mass flow rate

$$m = \rho AC$$

## Unit-4 Radiation

### 1. Define emissive power [E] and monochromatic emissive power. [ $E_{b\lambda}$ ]

The emissive power is defined as the total amount of radiation emitted by a body per unit time and unit area. It is expressed in  $W/m^2$ .

The energy emitted by the surface at a given length per unit time per unit area in all directions is known as monochromatic emissive power.

### 2. What is meant by absorptivity, reflectivity and transmissivity?

Absorptivity is defined as the ratio between radiation absorbed and incident radiation.

Reflectivity is defined as the ratio of radiation reflected to the incident radiation.

Transmissivity is defined as the ratio of radiation transmitted to the incident radiation.

### 3. What is black body and gray body?

Black body is an ideal surface having the following properties.

A black body absorbs all incident radiation, regardless of wave length and direction. For a prescribed temperature and wave length, no surface can emit more energy than black body.

If a body absorbs a definite percentage of incident radiation irrespective of their wave length, the body is known as gray body. The emissive power of a gray body is always less than that of the black body.

#### 4. State Planck's distribution law.

The relationship between the monochromatic emissive power of a black body and wave length of a radiation at a particular temperature is given by the following expression, by Planck.

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left(\frac{C_2}{\lambda T}\right)} - 1}$$

Where  $E_{b\lambda}$  = Monochromatic emissive power  $W/m^2$

$\lambda$  = Wave length – m

$C_1 = 0.374 \times 10^{-15} W m^2$

$C_2 = 14.4 \times 10^{-3} mK$

#### 5. State Wien's displacement law.

The Wien's law gives the relationship between temperature and wave length corresponding to the maximum spectral emissive power of the black body at that temperature.

$$\lambda_{\text{mas}} T = C_3$$

Where  $C_3 = 2.9 \times 10^{-3}$  [Radiation constant]

$$\Rightarrow \lambda_{\text{mas}} T = 2.9 \times 10^{-3} mK$$

#### 6. State Stefan – Boltzmann law. [April 2002, M.U.]

The emissive power of a black body is proportional to the fourth power of absolute temperature.

$$E_b \propto T^4$$

$$E_b = \sigma T^4$$

Where  $E_b$  = Emissive power,  $W/m^2$

$\sigma$  = Stefan. Boltzmann constant

$$= 5.67 \times 10^{-8} W/m^2 K^4$$

$T$  = Temperature, K

## 7. Define Emissivity.

It is defined as the ability of the surface of a body to radiate heat. It is also defined as the ratio of emissive power of any body to the emissive power of a black body of equal temperature.

$$\text{Emissivity } \varepsilon = \frac{E}{E_b}$$

## 8. State Kirchoff's law of radiation.

This law states that the ratio of total emissive power to the absorptivity is constant for all surfaces which are in thermal equilibrium with the surroundings. This can be written as

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3}$$

It also states that the emissivity of the body is always equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.

$$\alpha_1 = E_1; \alpha_2 = E_2 \text{ and so on.}$$

## 9. Define intensity of radiation ( $I_b$ ).

It is defined as the rate of energy leaving a space in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.

$$I_n = \frac{E_b}{\pi}$$

## 10. State Lambert's cosine law.

It states that the total emissive power  $E_b$  from a radiating plane surface in any direction is proportional to the cosine of the angle of emission

$$E_b \propto \cos \theta$$

## 11. What is the purpose of radiation shield?

Radiation shields constructed from low emissivity (high reflective) materials. It is used to reduce the net radiation transfer between two surfaces.

## 12. Define irradiation (G) and radiosity (J)

It is defined as the total radiation incident upon a surface per unit time per unit area. It is expressed in  $\text{W/m}^2$ .

It is used to indicate the total radiation leaving a surface per unit time per unit area. It is expressed in  $\text{W/m}^2$ .

### 13. What is meant by shape factor?

The shape factor is defined as the fraction of the radiative energy that is diffused from on surface element and strikes the other surface directly with no intervening reflections. It is represented by  $F_{ij}$ . Other names for radiation shape factor are view factor, angle factor and configuration factor.

#### Part-B

1. A black body at 3000 K emits radiation. Calculate the following:

- i) Monochromatic emissive power at  $7 \mu\text{m}$  wave length.
- ii) Wave length at which emission is maximum.
- iii) Maximum emissive power.
- iv) Total emissive power,
- v) Calculate the total emissive of the furnace if it is assumed as a real surface having emissivity equal to 0.85.

Given: Surface temperature  $T = 3000\text{K}$

Solution: 1. Monochromatic Emissive Power :

From Planck's distribution law, we know

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left(\frac{C_2}{\lambda T}\right)} - 1}$$

[From HMT data book, Page No.71]

Where

$$c_1 = 0.374 \times 10^{-15} \text{ W m}^2$$

$$c_2 = 14.4 \times 10^{-3} \text{ mK}$$

$$\lambda = 1 \times 10^{-6} \text{ m}$$

[Given]

$$\Rightarrow E_{b\lambda} = \frac{0.374 \times 10^{-15} [1 \times 10^{-6}]^{-5}}{\left[ \frac{14.4 \times 10^{-3}}{1 \times 10^{-6} \times 3000} \right] - 1}$$

$$E_{b\lambda} = 3.10 \times 10^{12} \text{ W/m}^2$$

### 2. Maximum wave length ( $\lambda_{\text{max}}$ )

From Wien's law, we know

$$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ mK}$$

$$\Rightarrow \lambda_{\max} = \frac{2.9 \times 10^{-3}}{3000}$$

$$\lambda_{\max} = 0.966 \times 10^{-6} \text{ m}$$

### 3. Maximum emissive power ( $E_{b\lambda}$ ) max:

Maximum emissive power

$$(E_{b\lambda})_{\max} = 1.307 \times 10^{-5} T^5$$

$$= 1.307 \times 10^{-5} \times (3000)^5$$

$$(E_{b\lambda})_{\max} = 3.17 \times 10^{12} \text{ W/m}^2$$

### 4. Total emissive power ( $E_b$ ):

From Stefan – Boltzmann law, we know that

$$E_b = \sigma T^4$$

[From HMT data book Page No.71]

Where  $\sigma$  = Stefan – Boltzmann constant  
 $= 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

$$\Rightarrow E_b = (5.67 \times 10^{-8}) (3000)^4$$

$$E_b = 4.59 \times 10^6 \text{ W/m}^2$$

### 5. Total emissive power of a real surface:

$$(E_b)_{\text{real}} = \epsilon \sigma T^4$$

Where  $\epsilon$  = Emissivity = 0.85

$$(E_b)_{\text{real}} = 0.85 \times 5.67 \times 10^{-8} \times (3000)^4$$

$$(E_b)_{\text{real}} = 3.90 \times 10^6 \text{ W/m}^2$$

2. Assuming sun to be black body emitting radiation at 6000 K at a mean distance of  $12 \times 10^{10} \text{ m}$  from the earth. The diameter of the sun is  $1.5 \times 10^9 \text{ m}$  and that of the earth is  $13.2 \times 10^6 \text{ m}$ . Calculate the following.

1. Total energy emitted by the sun.
2. The emission received per  $\text{m}^2$  just outside the earth's atmosphere.
3. The total energy received by the earth if no radiation is blocked by the earth's atmosphere.
4. The energy received by a  $2 \times 2 \text{ m}$  solar collector whose normal is inclined at  $45^\circ$  to the sun. The energy loss through the atmosphere is 50% and the diffuse radiation is 20% of direct radiation.

**Given:** Surface temperature  $T = 6000 \text{ K}$

Distance between earth and sun  $R = 12 \times 10^{10} \text{ m}$

Diameter on the sun  $D_1 = 1.5 \times 10^9 \text{ m}$

Diameter of the earth  $D_2 = 13.2 \times 10^6 \text{ m}$

**Solution:1. Energy emitted by sun  $E_b = \sigma T^4$**

$$\Rightarrow E_b = 5.67 \times 10^{-8} \times (6000)^4$$

$[\because \sigma = \text{Stefan - Boltzmann constant}]$   
 $= 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4]$

$$E_b = 73.4 \times 10^6 \text{ W/m}^2$$

Area of sun  $A_1 = 4\pi R_1^2$

$$= 4\pi \times \left( \frac{1.5 \times 10^9}{2} \right)^2$$

$A_1 = 7 \times 10^{18} \text{ m}^2$

$\Rightarrow$  Energy emitted by the sun

$$E_b = 73.4 \times 10^6 \times 7 \times 10^{18}$$

$E_b = 5.14 \times 10^{26} \text{ W}$

**2. The emission received per  $\text{m}^2$  just outside the earth's atmosphere:**

The distance between earth and sun  $R = 12 \times 10^{10} \text{ m}$

Area,  $A = 4\pi R^2$

$$= 4 \times \pi \times (12 \times 10^{10})^2$$

$A = 1.80 \times 10^{23} \text{ m}^2$

$\Rightarrow$  The radiation received outside the earth atmosphere per  $\text{m}^2$

$$= \frac{E_b}{A}$$
$$= \frac{5.14 \times 10^{26}}{1.80 \times 10^{23}}$$
$$= 2855.5 \text{ W/m}^2$$

**3. Energy received by the earth:**

$$\begin{aligned}\text{Earth area} &= \frac{\pi}{4}(D_2)^2 \\ &= \frac{\pi}{4} \times [13.2 \times 10^6]^2\end{aligned}$$

$$\boxed{\text{Earth area} = 1.36 \times 10^{14} \text{ m}^2}$$

Energy received by the earth

$$\begin{aligned}&= 2855.5 \times 1.36 \times 10^4 \\ &= 3.88 \times 10^{17} \text{ W}\end{aligned}$$

#### 4. The energy received by a 2 × 2 m solar collector;

Energy loss through the atmosphere is 50%. So energy reaching the earth.

$$\begin{aligned}&= 100 - 50 = 50\% \\ &= 0.50\end{aligned}$$

Energy received by the earth

$$\begin{aligned}&= 0.50 \times 2855.5 \\ &= 1427.7 \text{ W/m}^2 \quad \text{.....(1)}\end{aligned}$$

Diffuse radiation is 20%

$$\Rightarrow 0.20 \times 1427.7 = 285.5 \text{ W/m}^2$$

$$\boxed{\text{Diffuse radiation} = 285.5 \text{ W/m}^2} \quad \text{.....(2)}$$

Total radiation reaching the collection

$$\begin{aligned}&= 1427.7 + 285.5 \\ &= 1713.2 \text{ W/m}^2\end{aligned}$$

$$\begin{aligned}\text{Plate area} &= A \times \cos \theta \\ &= 2 \times 2 \times \cos 45^\circ \\ &= 2.82 \text{ m}^2\end{aligned}$$

Energy received by the collector

$$\begin{aligned}&= 2.82 \times 1713.2 \\ &= 4831.2 \text{ W}\end{aligned}$$

3. Two black square plates of size 2 by 2 m are placed parallel to each other at a distance of 0.5 m. One plate is maintained at a temperature of 1000°C and the other at 500°C. Find the heat exchange between the plates.

**Given:** Area  $A = 2 \times 2 = 4 \text{ m}^2$

$$T_1 = 1000^\circ\text{C} + 273$$

$$= 1273 \text{ K}$$

$$T_2 = 500^\circ\text{C} + 273$$

$$= 773 \text{ K}$$

$$\text{Distance} = 0.5 \text{ m}$$

**To find :** Heat transfer (Q)

**Solution :** We know Heat transfer general equation is

$$\text{where } Q_{12} = \frac{\sigma [T_1^4 - T_2^4]}{\frac{1 - \epsilon_1}{A_1 \epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \epsilon_2}{A_1 \epsilon_2}} \quad [\text{From equation No.(6)}]$$

For black body  $\epsilon_1 = \epsilon_2 = 1$

$$\Rightarrow Q_{12} = \sigma [T_1^4 - T_2^4] \times A_1 F_{12}$$

$$= 5.67 \times 10^{-8} [(1273)^4 - (773)^4] \times 4 \times F_{12}$$

$$Q_{12} = 5.14 \times 10^5 F_{12} \quad \dots\dots(1)$$

Where  $F_{12}$  – Shape factor for square plates

In order to find shape factor  $F_{12}$ , refer HMT data book, Page No.76.

$$X \text{ axis} = \frac{\text{Smaller side}}{\text{Distance between planes}}$$

$$= \frac{2}{0.5}$$

$$X \text{ axis} = 4$$

Curve  $\rightarrow 2$  [Since given is square plates]

X axis value is 4, curve is 2. So corresponding Y axis value is 0.62.

i.e.,  $F_{12} = 0.62$

(1)  $\Rightarrow Q_{12} = 5.14 \times 10^5 \times 0.62$

$$Q_{12} = 3.18 \times 10^5 \text{ W}$$

4. Two parallel plates of size 3 m × 2 m are placed parallel to each other at a distance of 1 m. One plate is maintained at a temperature of 550°C and the other at 250°C and the emissivities are 0.35 and 0.55 respectively. The plates are located in a large room whose walls are at 35°C. If the plates located exchange heat with each other and with the room, calculate.

1. Heat lost by the plates.

2. Heat received by the room.

**Given:** Size of the plates = 3 m × 2 m

Distance between plates = 1 m

First plate temperature  $T_1 = 550^\circ\text{C} + 273 = 823 \text{ K}$

Second plate temperature  $T_2 = 250^\circ\text{C} + 273 = 523 \text{ K}$

Emissivity of first plate  $\epsilon_1 = 0.35$

Emissivity of second plate  $\epsilon_2 = 0.55$

Room temperature  $T_3 = 35^\circ\text{C} + 273 = 308 \text{ K}$

**To find:** 1. Heat lost by the plates

2. Heat received by the room.

**Solution:** In this problem, heat exchange takes place between two plates and the room. So this is three surface problems and the corresponding radiation network is given below. Area

$$A_1 = 3 \times 2 = 6 \text{ m}^2$$

$$A_1 = A_2 = 6 \text{ m}^2$$

Since the room is large  $A_3 = \infty$

From electrical network diagram.

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-0.35}{0.35 \times 6} = 0.309$$

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.55}{0.55 \times 6} = 0.136$$

$$\frac{1-\varepsilon_3}{\varepsilon_3 A_3} = 0 \quad [\because A_3 = \infty]$$

Apply  $\frac{1-\varepsilon_3}{\varepsilon_3 A_3} = 0$ ,  $\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 0.309$ ,  $\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 0.136$  values in electrical network diagram.

To find shape factor  $F_{12}$  refer HMT data book, Page No.78.

$$X = \frac{b}{c} = \frac{3}{1} = 3$$

$$Y = \frac{a}{c} = \frac{2}{1} = 2$$

X value is 3, Y value is 2, corresponding shape factor [From table]

$$F_{12} = 0.47$$

$$\boxed{F_{12} = 0.47}$$

We know that,

$$F_{11} + F_{12} + F_{13} = 1 \quad \text{But,} \quad F_{11} = 0$$

$$\Rightarrow F_{13} = 1 - F_{12}$$

$$\Rightarrow F_{13} = 1 - 0.47$$

$$\boxed{F_{13} = 0.53}$$

$$\text{Similarly, } F_{21} + F_{22} + F_{23} = 1$$

We know

$$F_{22} = 0$$

$$\Rightarrow F_{23} = 1 - F_{21}$$

$$\Rightarrow F_{23} = 1 - F_{12}$$

$$F_{23} = 1 - 0.47$$

$$\boxed{F_{23} = 0.53}$$

From electrical network diagram,

$$\frac{1}{A_1 F_{13}} = \frac{1}{6 \times 0.53} = 0.314 \quad \dots(1)$$

$$\frac{1}{A_2 F_{23}} = \frac{1}{6 \times 0.53} = 0.314 \quad \dots(2)$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{6 \times 0.47} = 0.354 \quad \dots(3)$$

From Stefan – Boltzmann law, we know

$$E_b = \sigma T^4$$

$$E_{b1} = \sigma T_1^4$$

$$= 5.67 \times 10^{-8} [823]^4$$

$$\boxed{E_{b1} = 26.01 \times 10^3 \text{ W/m}^2} \quad \dots(4)$$

$$E_{b2} = \sigma T_2^4$$

$$= 5.67 \times 10^{-8} [823]^4$$

$$\boxed{E_{b2} = 4.24 \times 10^3 \text{ W/m}^2} \quad \dots(5)$$

$$E_{b3} = \sigma T_3^4$$

$$= 5.67 \times 10^{-8} [308]^4$$

$$\boxed{E_{b3} = J_3 = 510.25 \text{ W/m}^2} \quad \dots(6)$$

[From diagram]

The radiosities,  $J_1$  and  $J_2$  can be calculated by using Kirchoff's law.

⇒ The sum of current entering the node  $J_1$  is zero.

At Node  $J_1$ :

$$\frac{E_{b1} - J_1}{0.309} + \frac{J_2 - J_1}{\frac{1}{A_1 F_{12}}} + \frac{E_{b3} - J_1}{\frac{1}{A_1 F_{13}}} = 0$$

[From diagram]

$$\Rightarrow \frac{26.01 \times 10^3 - J_1}{0.309} + \frac{J_2 - J_1}{0.354} + \frac{510.25 - J_1}{0.314} = 0$$

$$\Rightarrow 84.17 \times 10^3 - \frac{J_1}{0.309} + \frac{J_2}{0.354} + \frac{J_1}{0.354} + 1625 - \frac{J_1}{0.354} = 0$$

$$\Rightarrow -9.24J_1 + 2.82J_2 = -85.79 \times 10^3 \quad \dots(7)$$

At node  $j_2$

$$\frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}} + \frac{E_{b3} - J_2}{\frac{1}{A_2 F_{23}}} + \frac{E_{b2} - J_2}{0.136} = 0 \quad \dots*$$

$$\frac{J_1 - J_2}{0.354} + \frac{510.25 - J_2}{0.314} + \frac{4.24 \times 10^3 - J_2}{0.136} = 0$$

$$\frac{J_1}{0.354} - \frac{J_2}{0.354} + \frac{510.25}{0.314} - \frac{J_2}{0.314} + \frac{4.24 \times 10^3}{0.136} - \frac{J_2}{0.136} = 0$$

$$\Rightarrow 2.82J_1 - 13.3J_2 = -32.8 \times 10^3 \quad \dots(8)$$

Solving equation (7) and (8),

$$\Rightarrow -9.24J_1 + 2.82J_2 = -85.79 \times 10^3 \quad \dots(7)$$

$$\Rightarrow 2.82J_1 - 13.3J_2 = -32.8 \times 10^3 \quad \dots(8)$$

$$\boxed{J_2 = 4.73 \times 10^3 \text{ W/m}^2}$$

$$\boxed{J_1 = 10.73 \times 10^3 \text{ W/m}^2}$$

Heat lost by plate (1) is given by

$$Q_1 = \frac{E_{b1} - J_1}{\left( \frac{1 - \epsilon_1}{\epsilon_1 A_1} \right)}$$

$$Q_1 = \frac{26.01 \times 10^3 - 10.73 \times 10^3}{\frac{1 - 0.35}{0.35 \times 6}}$$

$$\boxed{Q_1 = 49.36 \times 10^3 \text{ W}}$$

Heat lost by plate 2 is given by

$$Q_2 = \frac{E_{b2} - J_2}{\left( \frac{1 - \varepsilon_2}{\varepsilon_2 A_2} \right)}$$

$$Q_2 = \frac{4.24 \times 10^3 - 4.73 \times 10^3}{\frac{1 - 0.55}{6 \times 0.55}}$$

$$Q_2 = -3.59 \times 10^3 \text{ W}$$

Total heat lost by the plates

$$Q = Q_1 + Q_2$$

$$= 49.36 \times 10^3 - 3.59 \times 10^3$$

$$Q = 45.76 \times 10^3 \text{ W} \quad \dots\dots(9)$$

Heat received by the room

$$Q = \frac{J_1 - J_3}{\frac{1}{A_1 F_{13}}} + \frac{J_2 - J_3}{\frac{1}{A_1 F_{12}}}$$

$$= \frac{10.73 \times 10^3 - 510.25}{0.314} = \frac{4.24 \times 10^3 - 510.25}{0.314}$$

$$[\because E_{b1} = J_1 = 512.9]$$

$$Q = 45.9 \times 10^3 \text{ W} \quad \dots\dots(10)$$

From equation (9), (10), we came to know heat lost by the plates is equal to heat received by the room.

**5. A gas mixture contains 20% CO<sub>2</sub> and 10% H<sub>2</sub>O by volume. The total pressure is 2 atm. The temperature of the gas is 927°C. The mean beam length is 0.3 m. Calculate the emissivity of the mixture.**

**Given :** Partial pressure of CO<sub>2</sub>,  $P_{\text{CO}_2} = 20\% = 0.20 \text{ atm}$

Partial pressure of H<sub>2</sub>O,  $P_{\text{H}_2\text{O}} = 10\% = 0.10 \text{ atm}.$

Total pressure  $P = 2 \text{ atm}$

Temperature  $T = 927^\circ\text{C} + 273$

$= 1200 \text{ K}$

Mean beam length  $L_m = 0.3 \text{ m}$

**To find:** Emissivity of mixture ( $\epsilon_{\text{mix}}$ ).

**Solution : To find emissivity of  $\text{CO}_2$**

$$P_{\text{CO}_2} \times L_m = 0.2 \times 0.3$$

$$P_{\text{CO}_2} \times L_m = 0.06 \text{ m - atm}$$

From HMT data book, Page No.90, we can find emissivity of  $\text{CO}_2$ .

From graph, Emissivity of  $\text{CO}_2 = 0.09$

$$\epsilon_{\text{CO}_2} = 0.09$$

**To find correction factor for  $\text{CO}_2$**

Total pressure,  $P = 2 \text{ atm}$

$$P_{\text{CO}_2} L_m = 0.06 \text{ m - atm.}$$

From HMT data book, Page No.91, we can find correction factor for  $\text{CO}_2$

From graph, correction factor for  $\text{CO}_2$  is 1.25

$$C_{\text{CO}_2} = 1.25$$

$$\epsilon_{\text{CO}_2} \times C_{\text{CO}_2} = 0.09 \times 1.25$$

$$\epsilon_{\text{CO}_2} \times C_{\text{CO}_2} = 0.1125$$

**To find emissivity of  $\text{H}_2\text{O}$ :**

$$P_{\text{H}_2\text{O}} \times L_m = 0.1 \times 0.3$$

$$P_{H_2O} L_m = 0.03 \text{ m} \cdot \text{atm}$$

From HMT data book, Page No.92, we can find emissivity of  $H_2O$ .

From graph Emissivity of  $H_2O = 0.048$

$$\varepsilon_{H_2O} = 0.048$$

**To find correction factor for  $H_2O$ :**

$$\frac{P_{H_2O} + P}{2} = \frac{0.1 + 2}{2} = 1.05$$

$$\frac{P_{H_2O} + P}{2} = 1.05,$$

$$P_{H_2O} L_m = 0.03 \text{ m} \cdot \text{atm}$$

From HMT data book, Page No.92 we can find emission of  $H_2O$

**6. Two black square plates of size 2 by 2 m are placed parallel to each other at a distance of 0.5 m. One plate is maintained at a temperature of  $1000^\circ\text{C}$  and the other at  $500^\circ\text{C}$ . Find the heat exchange between the plates.**

**Given:** Area  $A = 2 \times 2 = 4 \text{ m}^2$

$$T_1 = 1000^\circ\text{C} + 273 = 1273 \text{ K}$$

$$T_2 = 500^\circ\text{C} + 273 = 773 \text{ K}$$

$$\text{Distance} = 0.5 \text{ m}$$

**To find :** Heat transfer (Q)

**Solution :** We know Heat transfer general equation is

$$\text{where } Q_{12} = \frac{\sigma [T_1^4 - T_2^4]}{\frac{1 - \varepsilon_1}{A_1 \varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \varepsilon_2}{A_1 \varepsilon_2}}$$

[From equation No.(6)]

For black body  $\varepsilon_1 = \varepsilon_2 = 1$

$$\Rightarrow Q_{12} = \sigma [T_1^4 - T_2^4] \times A_1 F_{12}$$

$$= 5.67 \times 10^{-8} [(1273)^4 - (773)^4] \times 4 \times F_{12}$$

$$\boxed{Q_{12} = 5.14 \times 10^5 F_{12}} \quad \dots\dots(1)$$

Where  $F_{12}$  – Shape factor for square plates

In order to find shape factor  $F_{12}$ , refer HMT data book, Page No.76.

$$X \text{ axis} = \frac{\text{Smaller side}}{\text{Distance between planes}}$$

$$= \frac{2}{0.5}$$

$$\boxed{X \text{ axis} = 4}$$

Curve  $\rightarrow 2$  [Since given is square plates]

X axis value is 4, curve is 2. So corresponding Y axis value is 0.62.

$$\text{i.e., } \boxed{F_{12} = 0.62}$$

$$(1) \Rightarrow Q_{12} = 5.14 \times 10^5 \times 0.62$$

$$\boxed{Q_{12} = 3.18 \times 10^5 \text{ W}}$$

From graph,

Correction factor for  $H_2O = 1.39$

$$\boxed{C_{H_2O} = 1.39}$$

$$\varepsilon_{H_2O} \times C_{H_2O} = 0.048 \times 1.39$$

$$\boxed{\varepsilon_{H_2O} \times C_{H_2O} = 0.066}$$

**Correction factor for mixture of  $CO_2$  and  $H_2O$ :**

$$\frac{P_{H_2O}}{P_{H_2O} + P_{CO_2}} = \frac{0.1}{0.1 + 0.2} = 1.05$$

$$\boxed{\frac{P_{H_2O}}{P_{H_2O} + P_{CO_2}} = 0.333}$$

$$P_{CO_2} \times L_m + P_{H_2O} \times L_m = 0.06 + 0.03$$

$$\boxed{P_{CO_2} \times L_m + P_{H_2O} \times L_m = 0.09}$$

From HMT data book, Page No.95, we can find correction factor for mixture of  $\text{CO}_2$  and  $\text{H}_2\text{O}$ .

## Unit-5 Mass Transfer

### 1. What is mass transfer?

The process of transfer of mass as a result of the species concentration difference in a mixture is known as mass transfer.

### 2. Give the examples of mass transfer.

Some examples of mass transfer.

1. Humidification of air in cooling tower
2. Evaporation of petrol in the carburetor of an IC engine.
3. The transfer of water vapour into dry air.

### 3. What are the modes of mass transfer?

There are basically two modes of mass transfer,

1. Diffusion mass transfer
2. Convective mass transfer

### 4. What is molecular diffusion?

The transport of water on a microscopic level as a result of diffusion from a region of higher concentration to a region of lower concentration in a mixture of liquids or gases is known as molecular diffusion.

### 5. What is Eddy diffusion?

When one of the diffusion fluids is in turbulent motion, eddy diffusion takes place.

### 6. What is convective mass transfer?

Convective mass transfer is a process of mass transfer that will occur between surface and a fluid medium when they are at different concentration.

### 7. State Fick's law of diffusion.

The diffusion rate is given by the Fick's law, which states that molar flux of an element per unit area is directly proportional to concentration gradient.

$$\frac{m_a}{A} = -D_{ab} \frac{dC_a}{dx}$$

where,

$$\frac{m_a}{A} \text{ -- Molar flux, } \frac{\text{kg -mole}}{\text{s-m}^2}$$

$D_{ab}$  Diffusion coefficient of species a and b,  $\text{m}^2/\text{s}$

$$\frac{dC_a}{dx} \text{ -- concentration gradient, kg/m}^3$$

### 8. What is free convective mass transfer?

If the fluid motion is produced due to change in density resulting from concentration gradients, the mode of mass transfer is said to be free or natural convective mass transfer.

Example : Evaporation of alcohol.

### 9. Define forced convective mass transfer.

If the fluid motion is artificially created by means of an external force like a blower or fan, that type of mass transfer is known as convective mass transfer.

Example: The evaporation of water from an ocean when air blows over it.

### 10. Define Schmidt Number.

It is defined as the ratio of the molecular diffusivity of momentum to the molecular diffusivity of mass.

$$Sc = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of mass}}$$

### 11. Define Sherwood Number.

It is defined as the ratio of concentration gradients at the boundary.

$$Sc = \frac{h_m x}{D_{ab}}$$

$h_m$  – Mass transfer coefficient,  $\text{m/s}$

$D_{ab}$  – Diffusion coefficient,  $\text{m}^2/\text{s}$

$x$  – Length,  $\text{m}$

Part-B

1. Hydrogen gases at 3 bar and 1 bar are separated by a plastic membrane having thickness 0.25 mm. the binary diffusion coefficient of hydrogen in the plastic is  $9.1 \times 10^{-3} \text{ m}^2/\text{s}$ . The solubility of hydrogen in the membrane is  $2.1 \times 10^{-3} \frac{\text{kg-mole}}{\text{m}^3 \text{ bar}}$

An uniform temperature condition of  $20^\circ$  is assumed.

Calculate the following

1. Molar concentration of hydrogen on both sides
2. Molar flux of hydrogen
3. Mass flux of hydrogen

**Given Data:**

Inside pressure  $P_1 = 3 \text{ bar}$

Outside pressure  $P_2 = 1 \text{ bar}$

Thickness,  $L = 0.25 \text{ mm} = 0.25 \times 10^{-3} \text{ m}$

Diffusion coefficient  $D_{ab} = 9.1 \times 10^{-8} \text{ m}^2/\text{s}$

Solubility of hydrogen  $= 2.1 \times 10^{-3} \frac{\text{kg-mole}}{\text{m}^3 \text{ - bar}}$

Temperature  $T = 20^\circ\text{C}$

**To find**

1. Molar concentration on both sides  $C_{a1}$  and  $C_{a2}$
2. Molar flux
3. Mass flux

**Solution :**

1. Molar concentration on inner side,

$C_{a1} = \text{Solubility} \times \text{inner pressure}$

$$C_{a1} = 2.1 \times 10^{-3} \times 3$$

$$C_{a1} = 6.3 \times 10^{-3} \frac{\text{kg - mole}}{\text{m}^3}$$

Molar concentration on outer side

$C_{a1} = \text{solubility} \times \text{Outer pressure}$

$$C_{a2} = 2.1 \times 10^{-3} \times 1$$

$$C_{a2} = 2.1 \times 10^{-3} \frac{\text{kg - mole}}{\text{m}^3}$$

2. We know  $\frac{m_o}{A} = \frac{D_{ab}}{L} [C_{a1} - C_{a2}]$

Molar flux,  $= \frac{9.1}{.25 \times 10^{-3}} - \frac{(6.3 \times 10^{-3} - 2.1 \times 10^{-3})}{.25 \times 10^{-3}} [1.2 - 0]$

$$\frac{m_a}{A} = 1.52 \times 10^{-6} \frac{\text{kg-mole}}{\text{s-m}^2}$$

3. Mass flux = Molar flux  $\times$  Molecular weight

$$= 1.52 \times 10^{-6} \frac{\text{kg-mole}}{\text{s-m}^2} \times 2 \text{ mole}$$

[ $\because$  Molecular weight of  $H_2$  is 2]

$$\text{Mass flux} = 3.04 \times 10^{-6} \frac{\text{kg}}{\text{s-m}^2}$$

**2. Oxygen at 25°C and pressure of 2 bar is flowing through a rubber pipe of inside diameter 25 mm and wall thickness 2.5 mm. The diffusivity of O<sub>2</sub> through rubber is  $0.21 \times 10^{-9} \text{ m}^2/\text{s}$  and the solubility of O<sub>2</sub> in rubber is  $3.12 \times 10^{-3} \frac{\text{kg-mole}}{\text{m}^3 \text{-bar}}$ . Find the loss of O<sub>2</sub> by diffusion per metre length of pipe.**

**Given data:**

Temperature,  $T = 25^\circ\text{C}$  fig

Inside pressure  $P_1 = 2 \text{ bar}$

Inner diameter  $d_1 = 25 \text{ mm}$

Inner radius  $r_1 = 12.5 \text{ mm} = 0.0125 \text{ m}$

Outer radius  $r_2 = \text{inner radius} + \text{Thickness}$

$$= 0.0125 + 0.0025$$

$$r_2 = 0.015 \text{ m}$$

Diffusion coefficient,  $D_{ab} = 0.21 \times 10^{-9} \text{ m}^2/\text{s}$

Solubility,  $= 3.12 \times 10^{-3} \frac{\text{kg-mole}}{\text{m}^3}$

Molar concentration on outer side,

$C_{a2} = \text{Solubility} \times \text{Outer pressure}$

$C_{a2} = 3.12 \times 10^{-3} \times 0$

$$C_{a2} = 0$$

[Assuming the partial pressure of  $O_2$  on the outer surface of the tube is zero]

We know,

$$\frac{m_a}{A} = \frac{D_{ab} [C_{a1} - C_{a2}]}{L}$$

$$\text{For cylinders, } L = r_2 - r_1; A = \frac{2\pi L (r_2 - r_1)}{\ln \left[ \frac{r_2}{r_1} \right]}$$

$$\begin{aligned} \text{Molar flux, (1)} &\Rightarrow \frac{m_a}{2\pi L (r_2 - r_1)} = \frac{D_{ab} [C_{a1} - C_{a2}]}{(r_2 - r_1)} \\ &\Rightarrow m_a = \frac{2\pi L D_{ab} [C_{a1} - C_{a2}]}{\ln \frac{r_2}{r_1}} \quad [\because \text{Length} = 1\text{m}] \\ m_a &= 4.51 \times 10^{-11} \frac{\text{kg-mole}}{\text{s}} \end{aligned}$$

3. An open pan 210 mm in diameter and 75 mm deep contains water at  $25^\circ\text{C}$  and is exposed to dry atmospheric air. Calculate the diffusion coefficient of water in air. Take the rate of diffusion of water vapour is  $8.52 \times 10^{-4} \text{ kg/h}$ .

**Given :**

$$\text{Diameter } d = 210 = .210 \text{ m}$$

$$\text{Deep } (x_2 - x_1) = 75 \text{ mm} = .075 \text{ m}$$

$$\text{Temperature, } T = 25^\circ\text{C} + 273 = 298\text{K}$$

$$\text{Diffusion rate (or) mass rate, } = 8.52 \times 10^{-4} \text{ kg/h}$$

$$= 8.52 \times 10^{-4} \text{ kg}/3600\text{s} = 2.36 \times 10^{-7} \text{ kg/s}$$

$$\text{Mass rate of water vapour} = 2.36 \times 10^{-7} \text{ kg/s}$$

**To find**

Diffusion coefficient ( $D_{ab}$ )

**Solution**

Dry atmospheric air

We know that, molar rate of water vapour.

$$\frac{m_a}{A} = \frac{D_{ab}}{GT} \frac{P}{(x_2 - x_1)} \times \ln \left[ \frac{P - P_{w2}}{P - P_{w1}} \right]$$

$$m_a = \frac{D_{ab} \times A}{GT} \frac{P}{(x_2 - x_1)} \times \ln \left[ \frac{P - P_{w2}}{P - P_{w1}} \right]$$

We know that,

Mass rate of water vapour = Molar rate of water vapour  $\times$  Molecular weight of steam

$$2.36 \times 10^{-7} = \frac{D_{ab} \times A}{GT} \times \frac{P}{(x_2 - x_1)} \times \ln \left[ \frac{P - P_{w2}}{P - P_{w1}} \right] \times 18 \dots (1)$$

where,

$$A - \text{Area} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.210)^2 = 0.0346 \text{ m}^2$$

$$G - \text{Universal gas constant} = 8314 \frac{1}{\text{kg-mole-k}}$$

$$P - \text{total pressure} = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$$

$P_{w1}$  – Partial pressure at the bottom of the test tube corresponding to saturation temperature 25°C

At 25°C

$$P_{w1} = 0.03166 \text{ bar}$$

$$P_{w1} = 0.03166 \times 10^5 \text{ N/m}^2$$

$P_{w2}$  = Partial pressure at the top of the pan, that is zero

$$P_{w2} = 0$$

$$(1) \Rightarrow 2.36 \times 10^{-7}$$

$$= \frac{D_{ab} \times 0.0346}{8314 \times 298} \times \frac{1 \times 10^5}{0.075} \times \ln \left[ \frac{1 \times 10^5 - 0}{1 \times 10^5 - 0.03166 \times 10^5} \right] \times 18$$

$$D_{ab} = 2.18 \times 10^{-5} \text{ m}^2/\text{s}.$$

**4. An open pan of 150 mm diameter and 75 mm deep contains water at 25°C and is exposed to atmospheric air at 25°C and 50% R.H. Calculate the evaporation rate of water in grams per hour.**

**Given :**

Diameter,  $d = 150\text{mm} = .150\text{m}$

Deep  $(x_2 - x_1) = 75\text{ mm} = .075\text{m}$

Temperature,  $T = 25 + 273 = 298\text{ K}$

Relative humidity = 50%

**To find**

Evaporation rate of water in grams per hour

**Solution:**

Diffusion coefficient ( $D_{ab}$ ) [water + air] at  $25^\circ\text{C}$

$$= 93 \times 10^{-3} \text{ m}^2 / \text{h}$$

$$\Rightarrow D_{ab} = \frac{93 \times 10^{-3}}{3600} \text{ m}^2 / \text{s}$$

$$\boxed{D_{ab} = 2.58 \times 10^{-5} \text{ m}^2 / \text{s}.}$$

Atmospheric air 50% RH (2)

We know that, for isothermal evaporation,

Molar flux,

$$\frac{m_a}{A} = \frac{D_{ab}}{GT} \frac{P}{(x_2 - x_1)} \ln \left[ \frac{P - P_{w2}}{P - P_{w1}} \right] \dots\dots(1)$$

where,

$$A - \text{Area} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (.150)^2$$

$$[\text{Area} = 0.0176 \text{ m}^2]$$

$$G - \text{Universal gas constant} = 8314 \frac{\text{J}}{\text{kg-mole-K}}$$

$$P - \text{Total pressure} = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$$

$P_{w1}$  – Partial pressure at the bottom of the test tube  
corresponding to saturation temperature  $25^\circ\text{C}$

At  $25^\circ\text{C}$

$$P_{w1} = 0.03166 \text{ bar}$$

$$P_{w1} = 0.03166 \times 10^5 \text{ N/m}^2$$

$P_{w2}$  = Partial pressure at the top of the test pan corresponding to 25°C and 50% relative humidity.

At 25°C

$$P_{w2} = 0.03166 \text{ bar} = 0.03166 \times 10^5 \times 0.50$$

$$P_{w2} = 0.03166 \times 10^5 \times 0.50$$

$$P_{w2} = 1583 \text{ N/m}^2$$

$$(1) \Rightarrow \frac{a}{0.0176}$$

$$= \frac{2.58 \times 10^{-5}}{8314 \times 298} \times \frac{1 \times 10^5}{0.075} \times \ln \left[ \frac{1 \times 10^5 - 1583}{1 \times 10^5 - 0.03166 \times 10^5} \right]$$

$$\text{Molar rate of water vapour, } m_a = 3.96 \times 10^{-9} \frac{\text{kg-mole}}{\text{s}}$$

Mass rate of water vapour = Molar rate of water vapour  $\times$  Molecular weight of steam

$$= 3.96 \times 10^{-9} \times 18$$

$$\text{Mass rate of water vapour} = 7.13 \times 10^{-8} \text{ kg/s.}$$

$$= 7.13 \times 10^{-8} \times \frac{1000\text{g}}{1/3600^h}$$

$$\text{Mass rate of water vapour} = 0.256 \text{ g/h}$$

If  $Re < 5 \times 10^5$ , flow is laminar

If  $Re > 5 \times 10^5$ , flow is turbulent

For laminar flow :

$$\text{Sherwood Number (Sh)} = 0.664 (Re)^{0.5} (Sc)^{0.333}$$

[From HMT data book, Page No.179]

$$\text{where, } Sc - \text{Schmidt Number} = \frac{\nu}{D_{ab}}$$

$D_{ab}$  – Diffusion coefficient

$$\text{Sherwood Number, } Sh = \frac{h_m x}{D_{ab}}$$

Where,  $h_m$  – Mass transfer coefficient – m/s

For Turbulent flow :

$$\text{Sherwood Number (Sh)} = [0.037 (Re)^{0.8} - 871] Sc^{0.333}$$

$$Sh = \frac{h_m x}{D_{ab}} \quad [\text{From HMT data book, Page No.180}]$$

### Solved Problems on Flat Plate.

**5. Air at 10°C with a velocity of 3 m/s flows over a flat plate. The plate is 0.3 m long. Calculate the mass transfer coefficient.**

**Given :**

Fluid temperature,  $T_\infty = 10^\circ\text{C}$

Velocity,  $U = 3 \text{ m/s}$

Length,  $x = 0.3 \text{ m}$

**To find:** Mass transfer coefficient ( $h_m$ )

**Solution:** Properties of air at 10°C [From HMT data book, Page No.22]

Kinematic viscosity,  $\nu = 14.16 \times 10^{-6} \text{ m}^2/\text{s}$

We know that,

$$\begin{aligned} \text{Reynolds Number, } Re &= \frac{Ux}{\nu} \\ &= \frac{3 \times 0.3}{14.16 \times 10^{-6}} \\ Re &= 0.63 \times 10^5 < 5 \times 10^5 \end{aligned}$$

Since,  $Re < 5 \times 10^5$ , flow is laminar

**For Laminar flow, flat plate,**

$$\text{Sherwood Number (Sh)} = 0.664 (Re)^{0.5} (Sc)^{0.333} \dots (1)$$

[From HMT data book, Page No.179]

Where,  $Sc$  – Schmidt Number =  $\frac{\nu}{D_{ab}}$  .....(2)

$D_{ab}$  – Diffusion coefficient (water+Air) at  $10^\circ\text{C} = 8^\circ\text{C}$

$$= 74.1 \times 10^{-3} \frac{\text{m}^2}{3600\text{s}}$$

$$D_{ab} = 2.50 \times 10^{-5} \text{m}^2 / \text{s}.$$

$$(2) \Rightarrow Sc = \frac{14.16 \times 10^{-6}}{2.05 \times 10^{-5}}$$

$$Sc = 0.637$$

Substitute  $Sc$ ,  $Re$  values in equation (1)

$$(1) \Rightarrow Sh = 0.664 (0.63 \times 10^5)^{0.5} (0.687)^{0.333}$$

$$Sh = 147$$

We know that,

$$\text{Sherwood Number, } Sh = \frac{h_m x}{D_{ab}}$$
$$\Rightarrow 147 = \frac{h_m \times 0.3}{2.05 \times 10^{-5}}$$

$$\text{Mass transfer coefficient, } h_m = .01 \text{ m/s}.$$